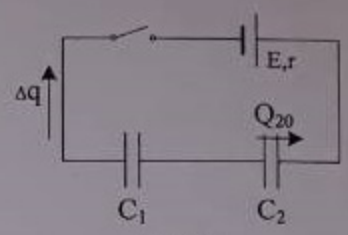
Петровић Катарина
DC 160069

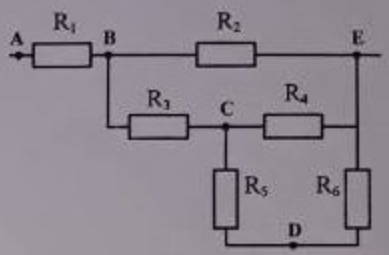
PISMENI ISPIT IZ ELEKTROTEHNIKE
14. jun 2018.

1. U kolu na Slici 1 poznato je $C_1 = C_2 = 40\,\text{mF}$, $E = 16\,\text{V}$, $r = 1\Omega$. Prekidač je otvoren, a kondenzator C_2 je opterećen početnom količinom naelektrisanja $Q_{20} = 80\,\text{mC}$. Odrediti količinu naelektrisanja Δq koja će proteći kroz kolo, nakon zatvaranja prekidača. Označiti referentne smerove i odrediti napon na kondenzatoru C_1 i količinu naelektrisanja na kondenzatoru C_2 nakon uspostavljanja stacionarnog stanja. **(20 poena)**



Slika 1

2. Na Slici 2 prikazana je grupa od šest jednakih otpornika, $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = R = 6\,\text{m}\Omega$. Odrediti ekvivalentnu otpornost između tačaka D i C. **(15 poena)**



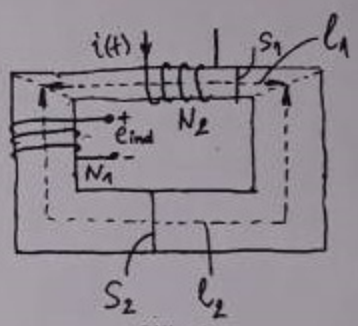
Slika 2

3. Na Slici 3 prikazano je magnetno kolo sa dva namotaja. Namotaj sa N_1 navojaka je otvorenih krajeva, a kroz namotaj sa N_2 navojaka protiče struja intenziteta $i(t) = I_m \cos \omega t$. Jezgro je homogeno i sastoji se iz dva dela površina poprečnog preseka S_1 i S_2, čije dužine srednjih linija iznose l_1 i l_2. Magnetna permeabilnost jezgra iznosi μ.

a) Odrediti intenzitet vektora jačine magnetnog polja u delu jezgra dužine l_2. **(10 poena)**

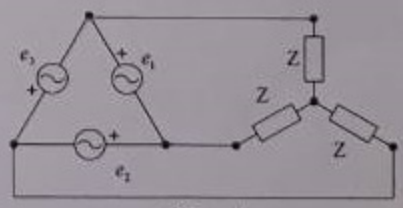
b) Odrediti sopstvenu induktivnost namotaja sa N_2 navojaka. **(5 poena)**

c) Odrediti efektivnu vrednost elektromotorne sile indukovane na krajevima namotaja sa N_1 navojaka. **(10 poena)**



Slika 3

4. Na Slici 4 prikazan je trofazni sistem generator-potrošač. Efektivna vrednost elektromotornih sila iznosi $E = 300\,\text{V}$, prividna snaga potrošača $S = 4.5\,\text{kVA}$, a reaktivna snaga $Q = -3.6\,\text{kvar}$. Odrediti efektivnu vrednost linijske struje i kompleksnu impedansu potrošača. **(20 poena)**

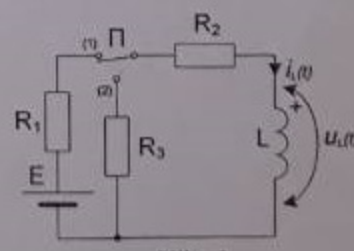


Slika 4

5. U kolu na Slici 5 poznate su vrednosti elemenata: $E = 24\,\text{V}$, $R_1 = R_2 = R_3 = 3\Omega$ i $L = 6\,\mu\text{H}$. Prekidač se nalazi u položaju (2) i u kolu je uspostavljeno stacionarno stanje. U trenutku $t = 0$, prekidač se prebacuje u položaj (1).

a) Odrediti izraze za struju i napon kalema nakon prebacivanja prekidača i nacrtati odgovarajuće vremenske dijagrame. **(15 poena)**

b) Odrediti u kom trenutku će struja kalema imati vrednost $i_L(t_x) = 3\,\text{A}$. **(5 poena)**



Slika 5

Електротехника

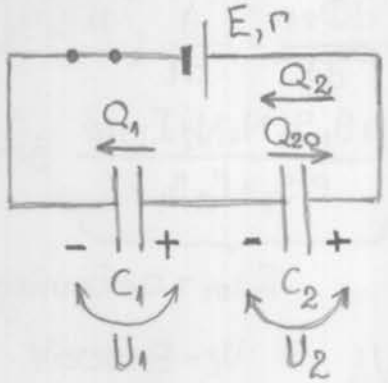
14.06.2018.

① $C_1 = C_2 = 40\ mF$, $E = 16\ V$, $r = 1\ \Omega$, $Q_{20} = 80\ mC$

$\Delta Q = ?$ $\quad C_1 = \frac{Q_1}{U_1} = \frac{\Delta Q}{U_1} \Rightarrow U_1 = \frac{\Delta Q}{C_1}$

$U_1 = ?$

$Q_2 = ?$ $\quad C_2 = \frac{Q_2}{U_2} = \frac{\Delta Q - Q_{20}}{U_2} \Rightarrow U_2 = \frac{\Delta Q - Q_{20}}{C_2}$



$U_1 + U_2 = E$

$\frac{\Delta Q}{C_1} + \frac{\Delta Q - Q_{20}}{C_2} = E$

$\frac{\Delta Q}{C_1} + \frac{\Delta Q}{C_2} - \frac{Q_{20}}{C_2} = E$

$\Delta Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = E + \frac{Q_{20}}{C_2}$

$$\boxed{\Delta Q = \frac{E + \frac{Q_{20}}{C_2}}{\frac{1}{C_1} + \frac{1}{C_2}} = 360\ mC}$$

$$\boxed{U_1 = \frac{\Delta Q}{C_1} = 9\ V}$$

$$\boxed{Q_2 = \Delta Q - Q_{20} = 280\ mC}$$

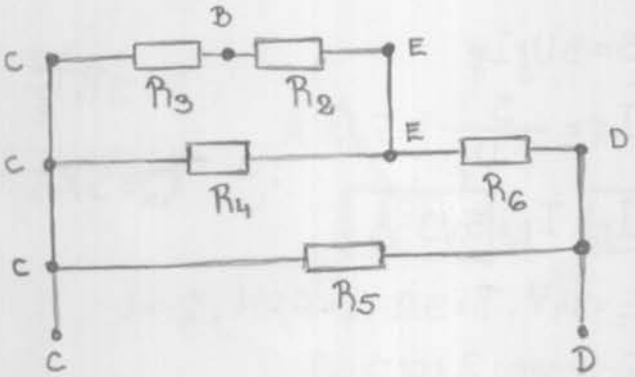
② $R_1 = R_2 = R_3 = R_4 = R_5 = R_6 = 6\ m\Omega$

$R_{DC} = ?$ $\quad R_{23} = R_2 + R_3 = 12\ m\Omega$

$R_{234} = \frac{R_{23} \cdot R_4}{R_{23} + R_4} = 4\ m\Omega$

$R_{2346} = R_{234} + R_6 = 10\ m\Omega$

$$\boxed{R_{DC} = \frac{R_{2346} \cdot R_5}{R_{2346} + R_5} = \frac{15}{4}\ m\Omega}$$



③ $N_1, I_1 = 0$

$N_2, i(t) = I_m \cos(\omega t)$

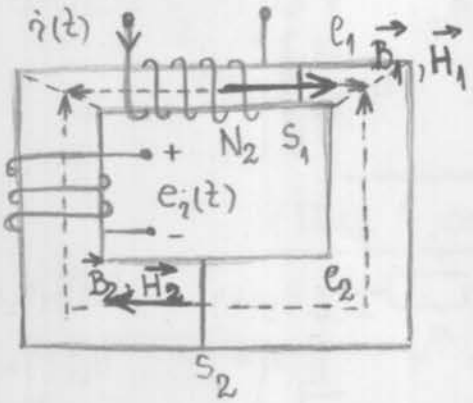
$S_1, S_2, \ell_1, \ell_2, \mu$

a) $H_2 = ?$ $\quad$ a) ЗОМФ: $\Phi = B_1 S_1 = B_2 S_2$

б) $L_2 = ?$ $\quad B_1 = B_2 \frac{S_2}{S_1}$

в) $E_i = ?$ $\quad$ АЗ: $H_1 \ell_1 + H_2 \ell_2 = N_2 i(t)$

$H_1 = \frac{B_1}{\mu} = \frac{B_2}{\mu} \frac{S_2}{S_1}$, $H_2 = \frac{B_2}{\mu}$



$\frac{B_2}{\mu} \frac{S_2}{S_1} \ell_1 + \frac{B_2}{\mu} \ell_2 = N_2 i(t)$

$\frac{B_2}{\mu}\left(\frac{S_2}{S_1} \ell_1 + \ell_2\right) = N_2 i(t)$

$\frac{B_2}{\mu}\left(\frac{\ell_1 S_2 + \ell_2 S_1}{S_1}\right) = N_2 I_m \cos(\omega t)$

$B_2 = \frac{\mu S_1 N_2 I_m \cos(\omega t)}{\ell_1 S_2 + \ell_2 S_1}$

$$\boxed{H_2 = \frac{B_2}{\mu} = \frac{S_1 N_2 I_m \cos(\omega t)}{\ell_1 S_2 + \ell_2 S_1}}$$

б) $\Phi_{S2} = N_2 B_1 S_1 = N_2 B_2 \frac{S_2}{S_1} S_1 = N_2 B_2 S_2$

$\Phi_{S2} = \frac{\mu S_1 S_2 N_2^2 I_m \cos(\omega t)}{\ell_1 S_2 + \ell_2 S_1}$

$$\boxed{L_2 = \frac{\Phi_{S2}}{i(t)} = \frac{\mu S_1 S_2 N_2^2}{\ell_1 S_2 + \ell_2 S_1}}$$

б) $\Phi_{21} = N_1 B_2 S_2 = \frac{\mu S_1 S_2 N_1 N_2 I_m \cos(\omega t)}{\ell_1 S_2 + \ell_2 S_1}$

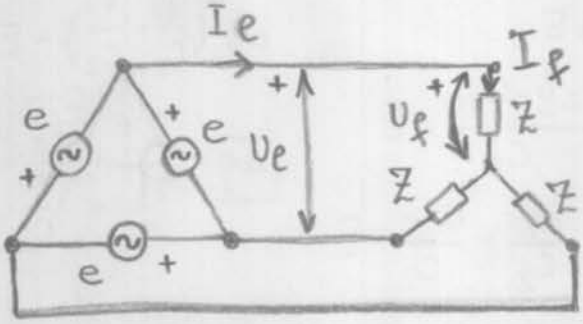
$$e_2(t) = -\frac{d\Phi_{21}}{dt} = -\frac{d}{dt}\left(\frac{\mu S_1 S_2 N_1 N_2 I_m \cos(\omega t)}{\ell_1 S_2 + \ell_2 S_1}\right) = -\frac{\mu S_1 S_2 N_1 N_2 I_m}{\ell_1 S_2 + \ell_2 S_1}(-\sin(\omega t))\,\omega$$

$$e_2(t) = \underbrace{\frac{\mu S_1 S_2 N_1 N_2 I_m \omega}{\ell_1 S_2 + \ell_2 S_1}}_{E_{2m}} \sin(\omega t), \quad \boxed{E_2 = \frac{E_{2m}}{\sqrt{2}} = \frac{\mu S_1 S_2 N_1 N_2 I_m \omega}{\sqrt{2}(\ell_1 S_2 + \ell_2 S_1)}}$$

E_{2m} - амплитуда ЕМС

④ $E = 300\,V$
$S = 4{,}5\,kVA$
$Q = 3{,}6\,kVAr$
$I_e = ?$
$\bar{Z} = ?$

$U_e = E = 300\,V$
$U_e = \sqrt{3}\,U_f \Rightarrow U_f = \frac{U_e}{\sqrt{3}}$
$U_f = 100\sqrt{3}\,V$
$P = \sqrt{S^2 - Q^2} = 2{,}7\,kW$



$S = 3U_f I_f$

$I_f = \frac{S}{3U_f} = 5\sqrt{3}\,A$

$\boxed{I_e = I_f = 5\sqrt{3}\,A}$

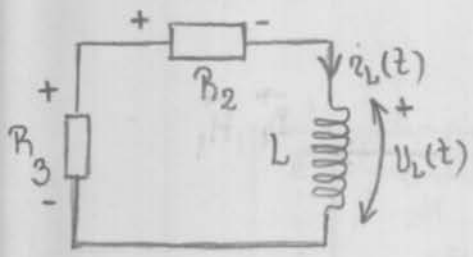
$P = 3RI_f^2 \Rightarrow R = \frac{P}{3I_f^2} = 12\,\Omega$

$Q = 3XI_f^2 \Rightarrow X = \frac{Q}{3I_f^2} = 16\,\Omega$

$\boxed{\bar{Z} = R + jX = 12 + j16\,[\Omega]}$

⑤ $E = 24\,V,\ R = 3\,\Omega,\ L = 6\,\mu H,\ 2 \to 1$

а) Положај 2 (СС):



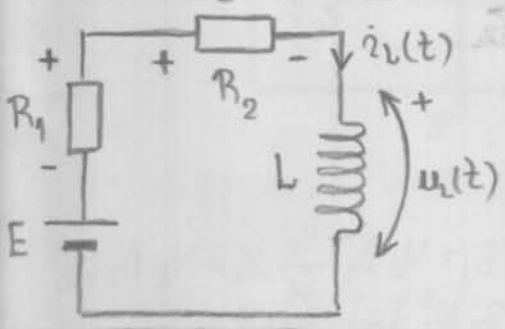
$i_L(t) = const.$
$u_L(t) = 0$

$-R_3 i_L(t) - R_2 i_L(t) - u_L(t) = 0$

$-i_L(t)(R_3 + R_2) = 0 \Rightarrow i_L(t) = 0,\ t \le 0$

$\boxed{i_L(0) = 0}$ почетни услов

Положај 1 (ПП):



$E - R_1 i_L(t) - R_2 i_L(t) - u_L(t) = 0 \,/(-1)$

$u_L(t) + (R_1 + R_2) i_L(t) = E,\ u_L(t) = L\frac{di_L(t)}{dt}$

$L\frac{di_L(t)}{dt} + (R_1 + R_2) i_L(t) = E \,/: L$

$\frac{di_L(t)}{dt} + \underbrace{\frac{R_1 + R_2}{L}}_{\alpha} i_L(t) = \underbrace{\frac{E}{L}}_{\beta}$

$\frac{\beta}{\alpha} = \frac{E}{R_1 + R_2} = 4A$

$K = ?\quad t = 0$

$i_L(0) = \frac{\beta}{\alpha} + K$

$K = i_L(0) - \frac{\beta}{\alpha} = -4A$

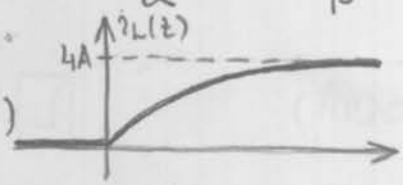
$\alpha = \frac{R_1 + R_2}{L} = 10^6\,\frac{1}{s}$

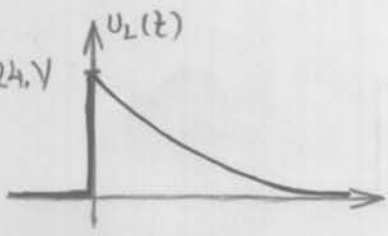
$\boxed{i_L(t) = 4 - 4e^{-10^6 t}\,[A],\ t \ge 0}$

$u_L(t) = L\frac{di_L(t)}{dt} = 6\cdot 10^{-6}\frac{d}{dt}(4 - 4e^{-10^6 t})$

$u_L(t) = 6\cdot 10^{-6}(-4\cdot e^{-10^6 t})(-10^6)$

$\boxed{u_L(t) = 24\cdot e^{-10^6 t}\,[V],\ t \ge 0}$





б) $i_L(t_x) = 3A,\ t_x = ?$

$i_L(t_x) = 4 - 4e^{-10^6 t_x} = 4(1 - e^{-10^6 t_x}) = 3$

$1 - e^{-10^6 t_x} = \frac{3}{4}$

$e^{-10^6 t_x} = \frac{1}{4} \,/\ln$

$-10^6 t_x = \ln\frac{1}{4} = \ln 4^{-1} = -\ln 4$

$\boxed{t_x = \frac{\ln 4}{10^6} = \ln 4 \cdot 10^{-6}\,s}$