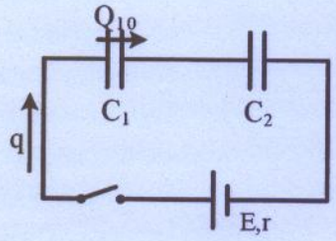
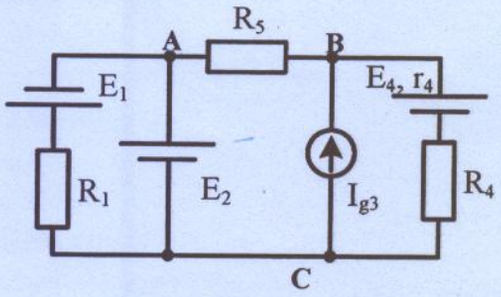
Elektrotehnika

pismeni ispit, 3.8.2025.

1. U kolu na Slici 1 prekidač je otvoren, kondenzator C_1 je opterećen početnom količinom naelektrisanja $Q_{10} = 10\ \mu C$, a kondenzator C_2 je neopterećen. Poznati su parametri elemenata: $E = 5\,V$, $r = 5\,\Omega$, $C_1 = 10\ \mu F$ i $C_2 = 10\ \mu F$. Odrediti proteklo naelektrisanje q po zatvaranju prekidača i elektrostatičke energije kondenzatora C_1 i C_2. **(10 poena)**



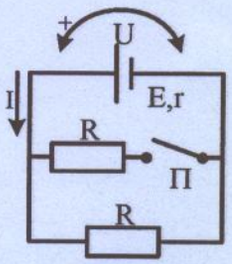
Slika 1



Slika 2

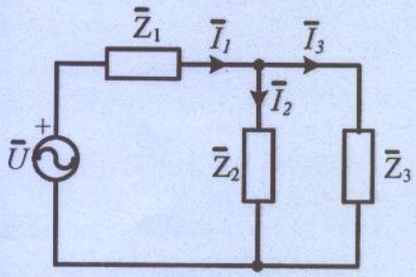
2. U kolu na Slici 2 poznate su vrednosti parametara elemenata: $R_1 = R_4 = R_5 = 10\ \Omega$, $E_1 = 20$ V, $E_2 = 10$ V, $E_4 = 20$ V, $r_4 = 5\ \Omega$ i $I_{g3} = 1$ A. Rešiti kolo primenom metode potencijala čvorova ili metode konturnih struja, a nakon toga odrediti snagu idealnog naponskog generator E_2. **(12 poena)**

3. Na Slici 3 poznata je vrednost otpornosti $R = 10\ \Omega$. Kada je prekidač **П** zatvoren, kroz naponski generator protiče struja $I = 3$ A. Kada je prekidač **П** otvoren, napon na krajevima realnog generatora je $U = 20$ V. Odrediti parametre realnog naposnkog generatora E i r. **(8 poena)**



Slika 3

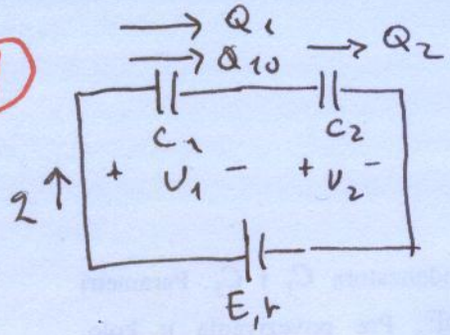
4. Na Slici 4 je prikazano kolo naizmenične struje koje čine naponski generator $u(t) = 100\sqrt{2}\cos(1000t)$ V i tri potrošača poznatih vrednosti kompleksnih impedansi:
$\overline{Z_1} = j10\ \Omega$, $\overline{Z_2} = (10 + j10)\ \Omega$ i $\overline{Z_3} = (10 - j10)\ \Omega$.
a) Odrediti kompleksne izraze za struje u svim granama kola. **(6 p)**
b) Nacrtati fazorski dijagram ovih struja. **(2 poena)**
c) Odrediti vremenski oblik struja $i_1(t)$ i $i_2(t)$. **(2 poena)**



Slika 4

5. Na naponski generator $U = 1000$ V, $f = 50$ Hz, priključena je paralelna veza pretežno kapacitivnog potrošača čija je aktivna snaga 50 kVA, a faktor snage $4/5$ i pretežno induktivnog potrošača kroz koji protiče struja efektivne vrednosti $30\sqrt{2}$A , a njegov faktor snage je $1/\sqrt{2}$. Odrediti:
a) ukupnu aktivnu, reaktivnu i prividnu snagu paralelne veze potrošača; **(6 poena)**
b) efektivnu vrednost struje koju paralelna veza potrošača uzima iz mreže i kompleksnu impedansu celokupnog potrošača. **(4 poena)**

1



$U_1 + U_2 - E = 0$

$\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = E$

$\frac{q+Q_{10}}{C_1} + \frac{q}{C_2} = E$

$q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) = E - \frac{Q_{10}}{C_1}$

$q = \frac{E - Q_{10}/C_1}{1/(C_1 + 1/C_2)} = \frac{5 - \frac{10\mu}{10\mu}}{\frac{1}{10\mu} + \frac{1}{10\mu}}$

$q = 20\mu C$

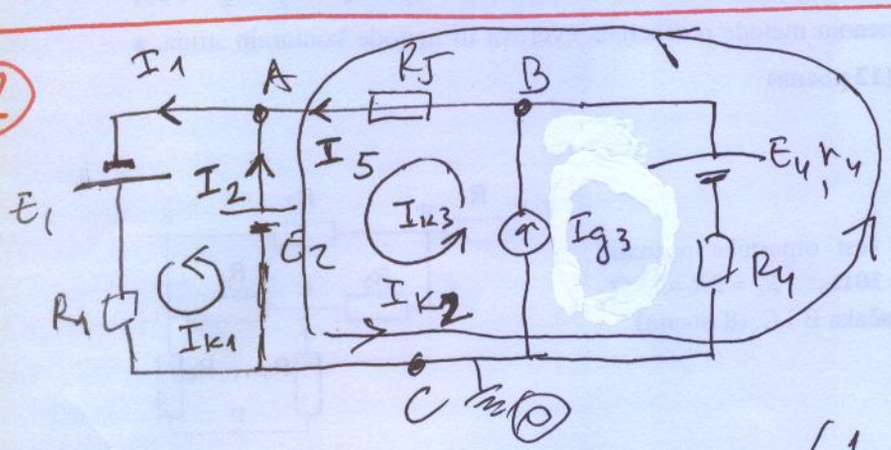
$Q_1 = q + Q_{10} = 30\mu C$

$Q_2 = q = 20\mu C$

$W_1 = \frac{1}{2}\frac{Q_1^2}{C_1} = \frac{90}{2}\mu J = 45\mu J$

$W_2 = \frac{1}{2}\frac{Q_2^2}{C_2} = \frac{400\mu^2}{2\cdot 10\mu} = 20\mu J$

2



MPC:

(A) $V_A = E_2 = 10V$

(B) $-\frac{1}{R_5}V_A + \left(\frac{1}{R_5} + \frac{1}{R_4 + r_4}\right)V_B = I_{g3} + \frac{E_4}{R_4 + r_4}$

$\left(\frac{1}{10} + \frac{1}{10+5}\right)V_B = 1 + \frac{20}{10+5} + \frac{10}{10}$

$\frac{5}{30}V_B = \frac{30+20}{15}$

$V_B = \frac{10}{3}\cdot 6 = 20V$

$I_2 = I_1 - I_5$

$I_1 = \frac{V_A + E_1}{R_1} = \frac{10+20}{10} = 3A$

$I_5 = \frac{U_{BA}}{R_5} = \frac{V_B - V_A}{R_5} = \frac{20-10}{10} = 1A$

$I_2 = 3 - 1 = 2A \Rightarrow P_{E2} = E_2 I_2 = 10\cdot 2 = 20W$

MKS:

(K1) $R_1 I_{K1} + 0\cdot I_{K2} + 0\cdot I_{K3} = E_1 + E_2$

(K2) $0\cdot I_{K1} + (R_4 + r_4 + R_5)I_{K2} + R_5 I_{K3} = E_4 - E_2$

(K3) $I_{K3} = I_{g3} = 1A$

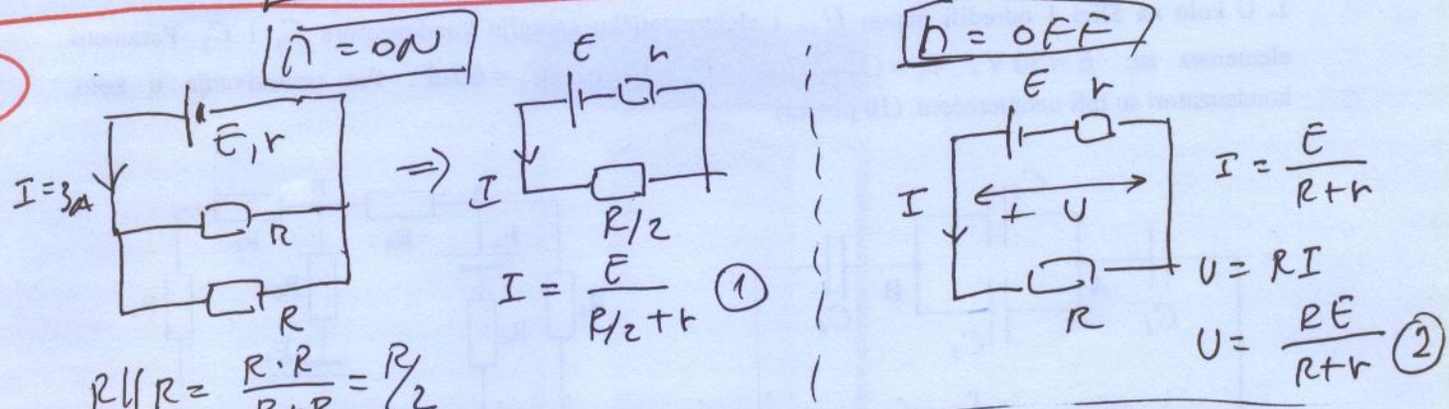
$I_{K1} = \frac{E_1 + E_2}{R_1} = \frac{20+10}{10} = 3A$

$I_{K2} = \frac{E_4 - E_2 - R_5 I_{g3}}{R_4 + r_4 + R_5} = \frac{20-10-10}{10+5+10} = \frac{0}{25} = 0A$

MKS nastavak: $I_2 = I_{K1} - I_{K2} - I_{K3} = 3 - 0 - 1 = 2A$

$$P_{E2} = E_2 I_2 = 20W$$

3



$$I = \frac{E}{R/2 + r} \quad (1)$$

$$R \| R = \frac{R \cdot R}{R + R} = R/2$$

$$I = \frac{E}{R + r}$$

$$U = RI$$

$$U = \frac{RE}{R + r} \quad (2)$$

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(1) $(R/2 + r)I = E \Rightarrow (5 + r) \cdot 3 = E \Rightarrow 3r - E = -15$ (1)

(2) $(R + r)U = RE \Rightarrow (10 + r) \cdot 20 = 10E \Rightarrow 20r - 10E = -200$ (2)

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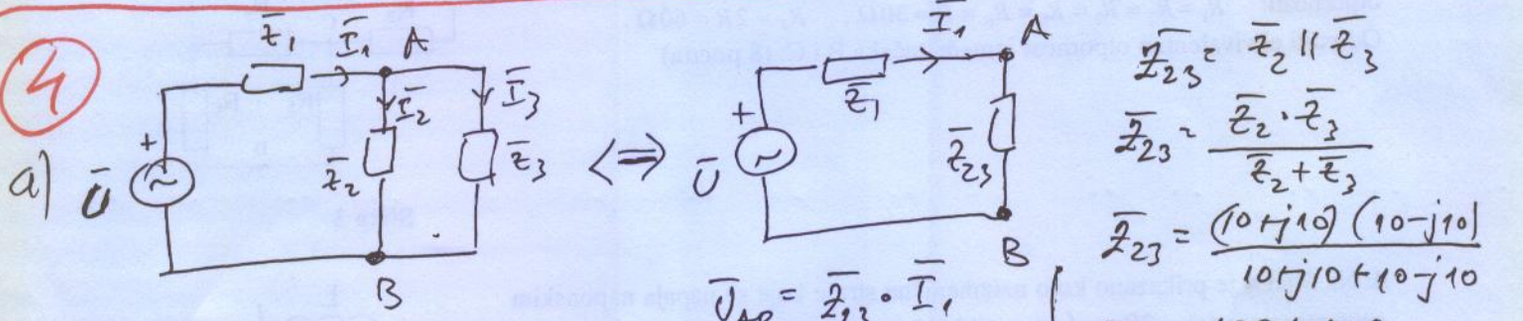
(2) $\Rightarrow r = E/2 - 10$ (3)

(3), (1) $\Rightarrow 3E/2 - 30 - E = -15 \Rightarrow E/2 = 15V \Rightarrow$ **E = 30V**

r = 5 Ω

4

a)



$$\bar{Z}_{23} = \bar{Z}_2 \| \bar{Z}_3$$

$$\bar{Z}_{23} = \frac{\bar{Z}_2 \cdot \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$\bar{Z}_{23} = \frac{(10 + j10)(10 - j10)}{10 + j10 + 10 - j10}$$

$$\bar{Z}_{23} = \frac{100 + 100}{20} = 10\,\Omega$$

$\bar{U} = 100\,V$

$\bar{U}_{AB} = \bar{Z}_{23} \cdot \bar{I}_1$

$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_1 + \bar{Z}_{23}}$

$$\bar{I}_1 = \frac{100}{j10 + 10} = \frac{100}{10\sqrt{2}\, e^{j\pi/4}} = \frac{10}{\sqrt{2}} e^{-j\pi/4} A$$

$$\bar{U}_{AB} = \frac{100}{\sqrt{2}} e^{-j\pi/4} V = 50\sqrt{2} e^{-j\pi/4} V$$

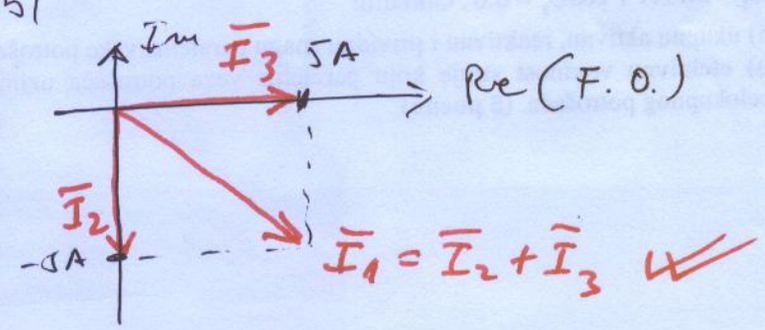
$$\bar{I}_2 = \frac{\bar{U}_{AB}}{\bar{Z}_2} = \frac{50\sqrt{2} e^{-j\pi/4}}{10\sqrt{2} e^{j\pi/4}}$$

$$\bar{I}_2 = 5 e^{-j\pi/2} A = -j5A$$

$$\bar{I}_3 = \frac{\bar{U}_{AB}}{\bar{Z}_3} = \frac{50\sqrt{2} e^{-j\pi/4}}{10\sqrt{2} e^{-j\pi/4}}$$

$$\bar{I}_3 = 5 e^{j0} A = 5A$$

b)

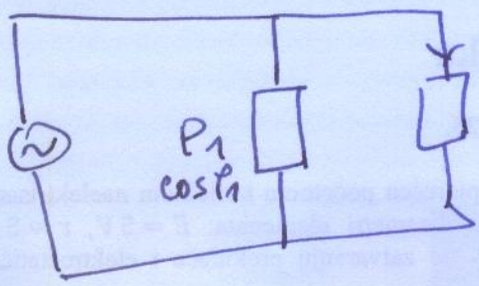


c)

$i_1(t) = 10\cos(1000t - \pi/4) A$

$i_2(t) = 5\sqrt{2}\cos(1000t - \pi/2) A$

5



a)

$I_2 = 30\sqrt{2}\ A$

$\cos\varphi_2 = \frac{1}{\sqrt{2}}$

$\varphi_2 > 0$ INDUKTIVNI

$\sin\varphi_2 = \sqrt{1-\cos^2\varphi_2}$

$\sin\varphi_2 = \sqrt{1-1/2} = \frac{1}{\sqrt{2}}$

$S_2 = U I_2 = 30\sqrt{2}\ KVA$

$P_2 = S_2 \cos\varphi_2 = 30\ kW$

$Q_2 = S_2 \sin\varphi_2 = 30\ KVAR$

$P_1 = 80\ kW$ TEKST ZADATKA NA ISPITU

$\cos\varphi_1 = 4/5$

$\varphi_1 < 0 \Rightarrow$ KAPACITIVNI

$\sin\varphi_1 = -\sqrt{1-\cos^2\varphi_1}$

$\sin\varphi_1 = -\sqrt{\frac{9}{25}} = -\frac{3}{5}$

$S_1 = \frac{50k}{\cos\varphi_1} = \frac{250}{4}\ KVA$

$S_1 = 62{,}5\ KVA$

$Q_1 = S_1 \sin\varphi_1 = -\frac{250}{4}\cdot\frac{3}{5}\ KVAR$

$Q_1 = -\frac{150}{4}\ KVAR = -37{,}5\ KVAR$

$P_{UK} = P_1 + P_2 = 80\ kW$

$Q_{UK} = Q_1 + Q_2 = 30k - \frac{75}{2}k = \frac{60k - 75k}{2} = -\frac{15}{2}\ KVAR$

$Q_{UK} = -7{,}5\ KVAR$

$S_{UK} = \sqrt{P_{UK}^2 + Q_{UK}^2} = \sqrt{80^2 + (-\frac{15}{2})^2} = \sqrt{6400 + \frac{225}{4}}$

$S_{UK} = \sqrt{\frac{25600+225}{4}} = \frac{\sqrt{25825}}{2}\ KVA \left(\approx 80{,}35\ KVA\right)$ UZ DIGITRON

NEOBAVEZNO

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b) $I_{UK} = \frac{S_{UK}}{U} = \frac{\sqrt{25825}}{2000}\ KA \left(\approx 80{,}35\right)\ A$ ← NEOBAV. UZ DIGITRON

$\bar{Z}_{UK} = R_{UK} + jX_{UK} \approx (12{,}39 + j\,1{,}16)\ \Omega$ NEOBAVEZNO UZ DIGITRON

$P_{UK} = R_{UK} I_{UK}^2 \Rightarrow R_{UK} = \frac{P_{UK}}{I_{UK}^2} = \frac{80.000}{\frac{25825}{4}} = \frac{320000}{25825}\ \Omega$

$Q_{UK} = X_{UK} I_{UK}^2 \Rightarrow X_{UK} = \frac{Q_{UK}}{I_{UK}^2} = -\frac{\frac{15000}{2}}{\frac{25825}{4^2}} = \frac{30.000}{25825}\ \Omega$