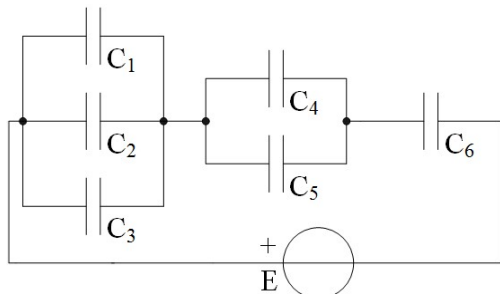


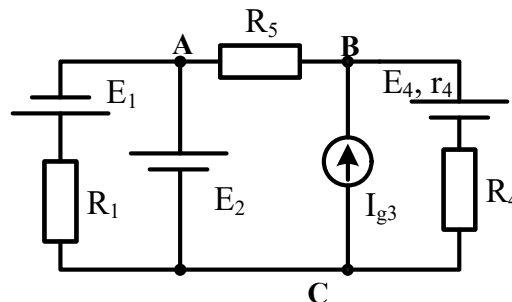
Elektrotehnika

pismeni ispit, 9.9.2025.

1. U kolu na Slici 1 poznate su kapacitivnosti $C_1 = C_2 = C_3 = C_4 = C = 6 \mu\text{F}$, $C_5 = 2C$, $C_6 = 3C$ i elektromotorna sila idealnog naponskog generatora $E = 15 \text{ V}$. Pre povezivanja, svi kondenzatori su bili neopterećeni. Odrediti naelektrisanje na C_6 , napon na C_4 i energiju kondenzatora C_5 . (8 poena)



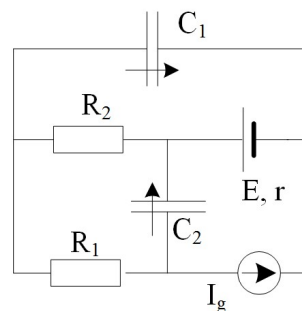
Slika 1



Slika 2

2. U kolu na Slici 2 poznate su vrednosti parametara elemenata: $R_1 = R_4 = R_5 = 10 \Omega$, $E_1 = 20 \text{ V}$, $E_2 = 10 \text{ V}$, $E_4 = 20 \text{ V}$, $r_4 = 5 \Omega$ i $I_{g3} = 1 \text{ A}$. Primenom Tevenenove teoreme odrediti intenzitet struje otpornika R_5 , a nakon toga i njegovu snagu. (12 poena)

3. Na Slici 3 poznato je $R_1 = R_2 = 10 \text{ k}\Omega$, $E = 20 \text{ V}$, $r = 5 \text{ k}\Omega$, $I_g = 1 \text{ mA}$ i $C_1 = C_2 = 22 \text{ nF}$. Odrediti količine naelektrisanja na kondenzatorima. (8 poena)



Slika 3

4. Na naponski generator $u(t) = 100 \cos(100t) \text{ V}$ priključena je paralelna veza idealnog kalema induktivnosti $L = 20 \text{ mH}$ i složenog potrošača kompleksne snage $\bar{S} = 1.5 - j0.5 \text{ kVA}$.

- Odrediti kompleksne izraze za struje u svim granama kola. (6 poena)
- Skicirati fazorski dijagram svih struja u kolu. (2 poena)
- Odrediti trenutnu vrednost struje kalema. (2 poena)
- Odrediti aktivnu, reaktivnu i prividnu snagu paralelne veze elemenata. (4 poena)

5. Na Slici 4 je prikazan trofazni sistem generator - potrošač. Ems generatora iznose:

$$e_1(t) = 400 \cdot \cos(314t + \pi/2) \text{ V}$$

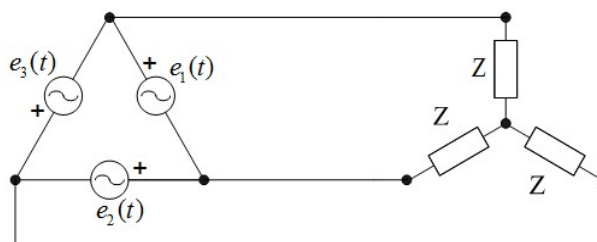
$$e_2(t) = 400 \cdot \cos(314t + 7\pi/6) \text{ V}$$

$$e_3(t) = 400 \cdot \cos(314t - \pi/6) \text{ V}, \text{ dok impedansa}$$

jedne faze potrošača iznosi $\bar{Z} = 20 - j10 \Omega$.

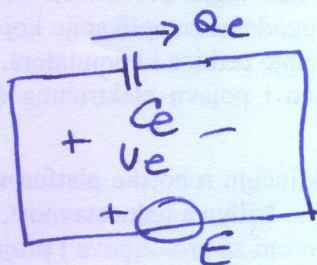
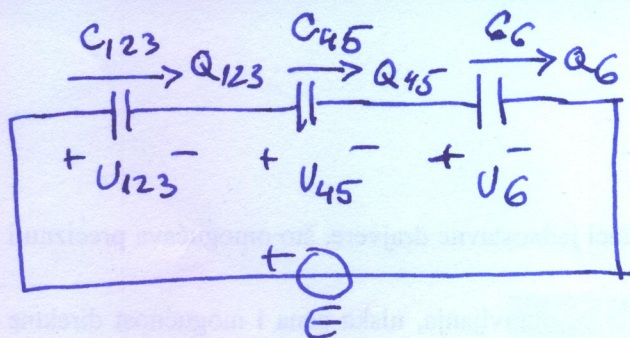
a) Nacrtati fazorski dijagram linijskih napona. (2 poena)

b) Odrediti efektivnu vrednost linijskih struja, prividnu i reaktivnu snagu potrošača. (6 poena)



Slika 4

①



$$U_e = E = 15V$$

$$Q_e = C_e U_e = CE = 90\mu C$$

$$Q_{123} = Q_{45} = Q_6 = Q_e = 90\mu C$$

$$C_{123} = C_1 + C_2 + C_3 = 3C = 18\mu F$$

$$C_{45} = C_4 + C_5 = 3C = 18\mu F$$

$$C_e = \frac{1}{\frac{1}{C_{123}} + \frac{1}{C_{45}} + \frac{1}{C_6}}$$

$$C_e = \frac{1}{\frac{1}{3C} + \frac{1}{3C} + \frac{1}{3C}} = \frac{1}{\frac{3}{3C}} = C$$

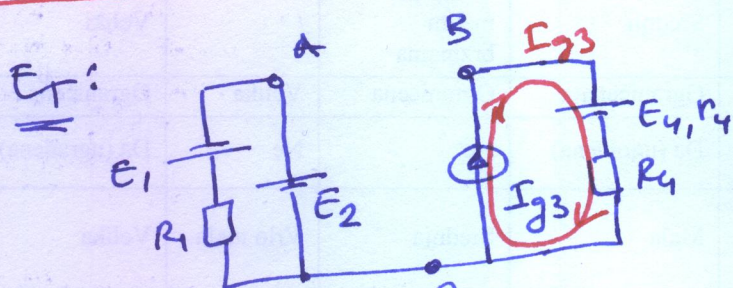
$$C_e = C = 6\mu F$$

$$U_4 = U_5 = U_{45} = \frac{Q_{45}}{C_{45}} = \frac{90\mu C}{18\mu F} = 5V$$

$$W_5 = \frac{1}{2} C_5 U_5^2 = \frac{1}{2} 2.6\mu \cdot 5^2$$

$$W_5 = 150\mu J$$

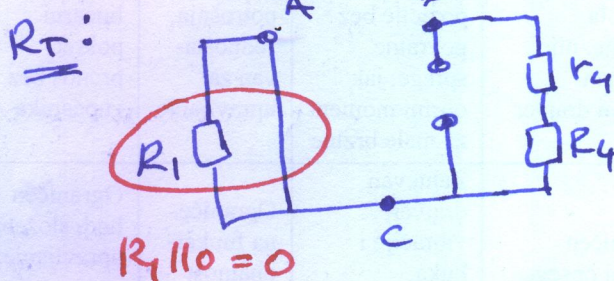
②



$$E_T = U_{AB}^{ov} = E_2 - R_4 I_{g3} - r_4 I_{g3} - E_4$$

$$E_T = 10 - 10 \cdot 1 - 5 \cdot 1 - 20$$

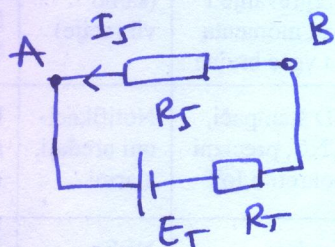
$$E_T = -25V$$



$$R_1 || 0 = 0$$

$$R_T = R_{AB} = (R_1 || 0) + (r_4 + R_4)$$

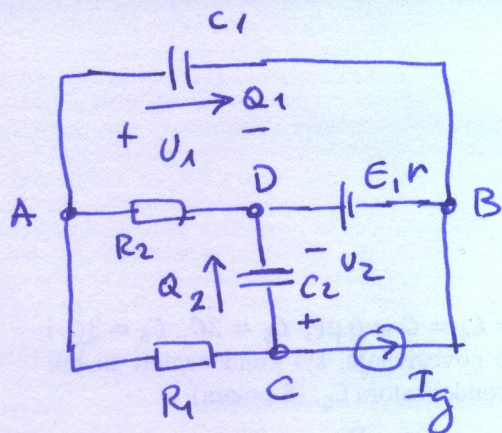
$$R_T = R_4 + r_4 = 10 + 5 = 15\Omega$$



$$I_5 = \frac{-E_T}{R_5 + R_T} = \frac{25}{10 + 15} = 1A$$

$$P_5 = R_5 \cdot I_5^2 = 10 \cdot 1^2 = 10W$$

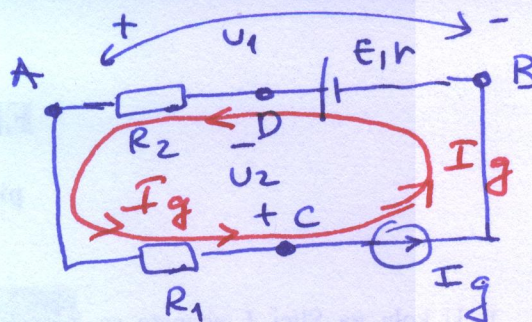
3



$$\Rightarrow$$

$$I_{C1} = 0$$

$$I_{C2} = 0$$



$$Q_1 = C_1 U_1 = C_1 U_{AB}$$

$$Q_1 = 22 \mu \cdot 5 = 110 \mu C$$

$$Q_2 = C_2 U_2 = C_2 U_{CD}$$

$$Q_2 = 22 \mu \cdot (-20) = -440 \mu C$$

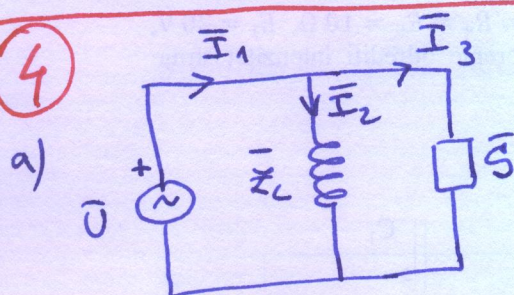
$$U_1 = U_{AB} = -(R_2 + r) I_g + E$$

$$U_1 = U_{AB} = 5 V$$

$$U_2 = U_{CD} = -(R_1 + R_2) I_g$$

$$U_2 = U_{CD} = -20 V$$

4



$$\bar{Z}_L = j\omega L = j100 \cdot 20 m$$

$$\bar{Z}_L = j2 \Omega$$

$$\bar{U} = \frac{100}{\sqrt{2}} e^{j0} = 50\sqrt{2} V$$

$$\bar{S} = (1.5 - j0.5) KVA = (1500 - j500) VA$$

$$\bar{S} = \bar{U} \bar{I}_3^* \Rightarrow \bar{I}_3^* = \frac{\bar{S}}{\bar{U}} \Rightarrow \bar{I}_3 = \bar{I}_3^{**} = \frac{\bar{S}^*}{\bar{U}^*}$$

$$\bar{I}_3 = \left(\frac{1500 - j500}{50\sqrt{2}} \right)^* = \frac{30}{\sqrt{2}} + j\frac{10}{\sqrt{2}} = (15\sqrt{2} + j5\sqrt{2}) A$$

$$\bar{I}_2 = \frac{\bar{U}}{\bar{Z}_L} = \frac{50\sqrt{2}}{j2} = -j25\sqrt{2} A$$

$$\bar{I}_1 = \bar{I}_2 + \bar{I}_3 = 15\sqrt{2} + j5\sqrt{2} - j25\sqrt{2} = (15\sqrt{2} - j20\sqrt{2}) A$$

$$c) \bar{I}_L = \bar{I}_2 = -j25\sqrt{2} = 25\sqrt{2} e^{-j\pi/2} A$$

$$I_L = 25\sqrt{2} A$$

$$I_{me} = I_L \sqrt{2} = 50 A$$

$$\psi = -\pi/2 \text{ rad}$$

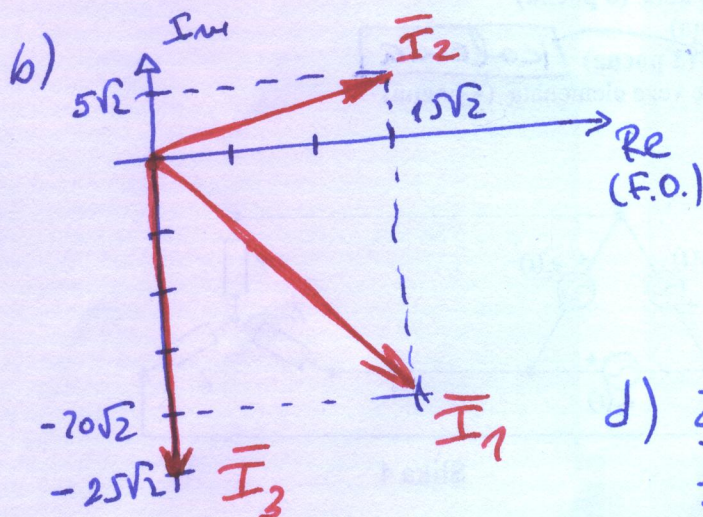
$$i_L(t) = i_2(t) = 50 \cos(100t - \pi/2) A$$

$$d) \bar{S}_{uk} = \bar{U} \bar{I}_1^* = 50\sqrt{2} (15\sqrt{2} + j20\sqrt{2})$$

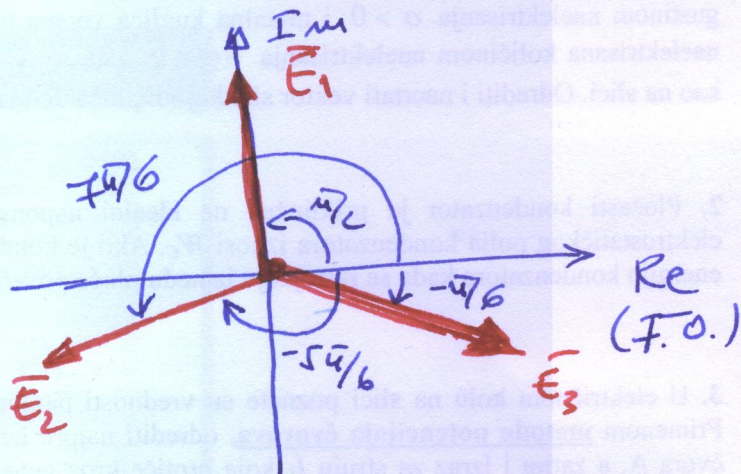
$$\bar{S}_{uk} = (1500 + j2000) VA$$

$$P = 1500 W \quad Q = 2000 VAR$$

$$S = \sqrt{P^2 + Q^2} = 500 \sqrt{3^2 + 4^2} = 2500 VA$$



5) a) $\bar{E}_1 = \frac{400}{\sqrt{2}} e^{j\pi/2} V = 200\sqrt{2} e^{j\pi/2} V$
 $\bar{E}_2 = \frac{400}{\sqrt{2}} e^{j\pi/6} V = 200\sqrt{2} e^{j\pi/6} V = 200\sqrt{2} e^{-j5\pi/6} V$
 $\bar{E}_3 = \frac{400}{\sqrt{2}} e^{-j\pi/6} V = 200\sqrt{2} e^{-j\pi/6} V$



b) $U_L = E_1 = E_2 = E_3 = 200\sqrt{2} V$

$$U_F = \frac{U_L}{\sqrt{3}} = 200\sqrt{2/3} V$$

$$Z = |\bar{Z}| = 10\sqrt{2^2 + 12} = 10\sqrt{15} \Omega$$

$$I_F = \frac{U_F}{Z} = \frac{200\sqrt{2/3}}{10\sqrt{15}} = 20\sqrt{\frac{2}{15}} A$$

$$I_L = I_F = 20\sqrt{2/15} A$$

$$(\bar{Z} = R + jX) \Rightarrow Q = 3 \times I_F^2 = 3 \cdot (-10) \cdot 400 \cdot \frac{2}{15} = -1600 \text{ VAR}$$

$$S = 3 \bar{Z} I_F^2 = 3 \cdot 10\sqrt{15} \cdot 400 \cdot \frac{2}{15} = 1600\sqrt{15} \text{ VA}$$