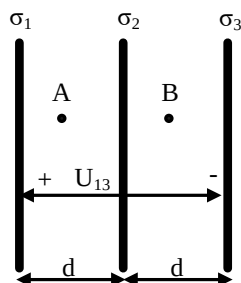


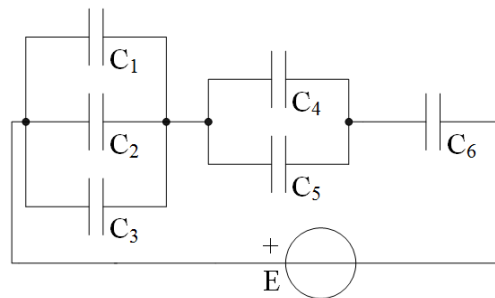
Elektrotehnika

pismeni ispit, 13.10.2025.

1. Tri veoma velike ravnomerno naelektrisanе metalne ploče nalaze se u sredini čija je dielektrična konstanta $\epsilon = 10^{-9}$ F/m, na međusobnom rastojanju od $d = 10$ m kao što je prikazano na Slici 1. Poznate su površinske gustine naelektrisanja ploča $\sigma_1 = -40$ nC/m², $\sigma_2 = 20$ nC/m² i $\sigma_3 = -60$ nC/m². Odrediti intenzitet i skicirati vektor električnog polja u tačkama A i B, a zatim odrediti napon U_{13} između prve i treće ploče. (8 poena)



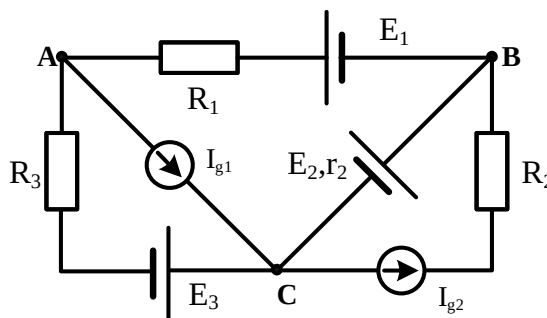
Slika 1



Slika 2

2. U kolu na Slici 2 poznate su kapacitivnosti $C_1 = C_4 = C = 6$ μ F, $C_2 = C_5 = 2C$, $C_3 = C_6 = 3C$, i elektromotorna sila idealnog naponskog generatora $E = 20$ V. Pre povezivanja, svi kondenzatori su bili neopterećeni. Odrediti naelektrisanje na C_4 , napon na C_1 i energiju kondenzatora C_6 . (8 poena)

3. U kolu na Slici 3 poznate su vrednosti parametara elemenata: $R_1 = R_2 = R_3 = 10$ Ω , $E_1 = 10$ V, $E_2 = 40$ V, $E_3 = 30$ V, $r_2 = 10$ Ω , $I_{g1} = 4$ A i $I_{g2} = 3$ A. Rešiti kolo primenom metode potencijala čvorova ili metode konturnih struja, a nakon toga odrediti snagu generatora E_1 . (10 poena)



Slika 3

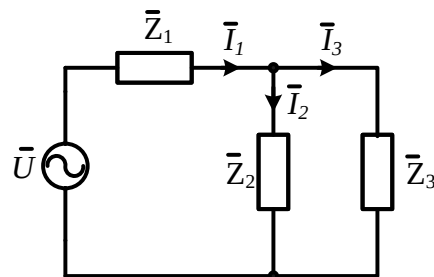
4. Na Slici 4 je prikazano kolo naizmenične struje koje čine naponski generator $u(t) = 100 \cos(\omega t - \pi/4)$ V i tri potrošača poznatih vrednosti kompleksnih impedansi: $\bar{Z}_1 = -j20$ Ω , $\bar{Z}_2 = j20$ Ω i $\bar{Z}_3 = 20$ Ω .

a) Odrediti kompleksne izraze za struje u svim granama kola.

(6 poena)

b) Odrediti trenutnu vrednost napona na impedansi Z_1 . (2 poena)

c) Odrediti aktivnu, reaktivnu i prividnu snagu i faktor snage paralelne veze Z_2 i Z_3 . (4 poena)



Slika 4

5. Na naponski generator efektivne vrednosti $U = 100$ V, priključena je paralelna veza pretežno induktivnog potrošača, konduktanse 0.03 S i faktora snage $1/\sqrt{2}$, i jednog idealnog kalema prividne snage 100 VA. Odrediti:

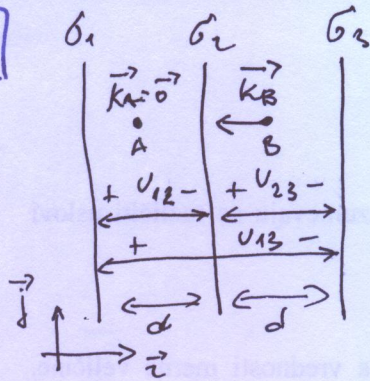
a) efektivne vrednosti struja pojedinačnih potrošača; (3 poena)

b) ukupnu aktivnu, reaktivnu i prividnu snagu celokupnog potrošača (paralelne veze); (5 poena)

c) efektivnu vrednost struje koju celokupni potrošač uzima iz generatora; (2 poena)

d) kompleksnu admitansu celokupnog potrošača. (2 poena)

1



$$\vec{K}_A = \vec{K}_{A1} + \vec{K}_{A2} + \vec{K}_{A3}$$

$$\vec{K}_A = \frac{\sigma_1}{2\epsilon} \vec{i} + \frac{\sigma_2}{2\epsilon} (-\vec{i}) + \frac{\sigma_3}{2\epsilon} (-\vec{i})$$

$$\vec{K}_A = \frac{\sigma_1 - \sigma_2 - \sigma_3}{2\epsilon} \vec{i} = \frac{(-40 - 20 + 60) \cdot 10^{-9}}{2 \cdot 10^{-9}} \vec{i} = \frac{0}{2} \vec{i} = 0 \frac{V}{m}$$

$$\vec{K}_A = 0 \vec{i} \frac{V}{m}$$

$$\vec{K}_B = \vec{K}_{B1} + \vec{K}_{B2} + \vec{K}_{B3}$$

$$\vec{K}_B = \frac{\sigma_1}{2\epsilon} \vec{i} + \frac{\sigma_2}{2\epsilon} \vec{i} + \frac{\sigma_3}{2\epsilon} (-\vec{i})$$

$$\vec{K}_B = \frac{\sigma_1 + \sigma_2 - \sigma_3}{2\epsilon} \vec{i} = \frac{(-40 + 20 + 60) \cdot 10^{-9}}{2 \cdot 10^{-9}} \vec{i} = \frac{40}{2} \vec{i} = 20 \vec{i} \frac{V}{m}$$

$$\vec{K}_B = 20 \vec{i} \frac{V}{m}$$

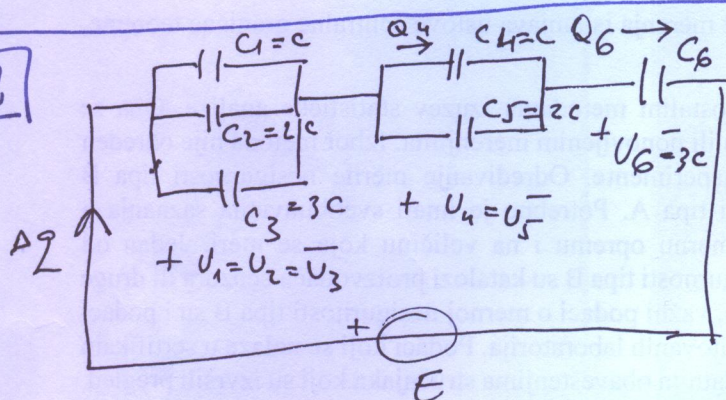
$$U_{13} = U_{12} + U_{23}$$

$$U_{12} = |\vec{K}_A| \cdot d = 0V$$

$$U_{23} = |\vec{K}_B| \cdot d = 200V$$

$$U_{13} = 0 + 200 = 200V$$

2



$$Q_6 = \Delta Q = 86.4 \mu C$$

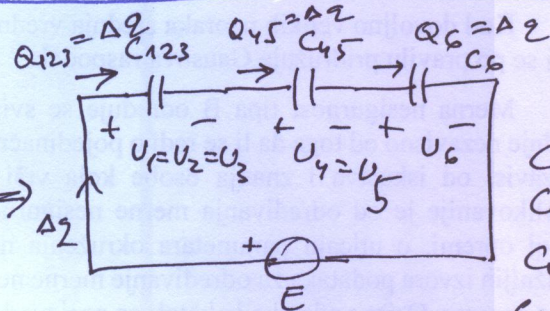
$$U_6 = \frac{Q_6}{C_6} = \frac{86.4 \mu C}{3 \mu F} = \frac{2}{5} E = 8V$$

$$U_4 = U_5 = \frac{Q_{45}}{C_{45}} = \frac{5 \mu C}{3 \mu F} = \frac{2}{5} E = 8V$$

$$U_1 = U_2 = U_3 = \frac{Q_{123}}{C_{123}} = \frac{\Delta Q}{6 \mu F} = \frac{1}{5} E = 4V$$

$$Q_4 = C_4 U_4 = 1 \mu F \cdot \frac{2}{5} E = \frac{2}{5} \mu C = 48 \mu C$$

$$W_6 = \frac{1}{2} C_6 U_6^2 = \frac{1}{2} 3 \mu F \cdot \left(\frac{2E}{5}\right)^2 = \frac{6 \mu C E^2}{25} = 576 \mu J$$



$$C_{123} = C_1 + C_2 + C_3$$

$$C_{123} = 6 \mu F$$

$$C_{123} = 36 \mu F$$

$$C_{45} = C_4 + C_5$$

$$C_{45} = 3 \mu F$$

$$C_{45} = 18 \mu F$$

$$E = U_1 + U_4 + U_6$$

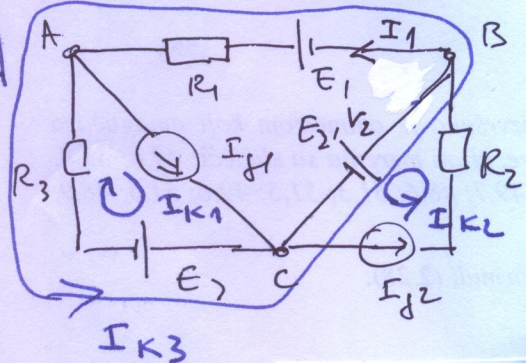
$$E = \frac{\Delta Q}{C_{123}} + \frac{\Delta Q}{C_{45}} + \frac{\Delta Q}{C_6}$$

$$E = \left(\frac{1}{C_{123}} + \frac{1}{C_{45}} + \frac{1}{C_6} \right) \Delta Q$$

$$E = \left(\frac{1}{6 \mu F} + \frac{1}{3 \mu F} + \frac{1}{3 \mu F} \right) \Delta Q = \frac{5 \Delta Q}{6 \mu F}$$

$$\Delta Q = \frac{6}{5} \mu C = 144 \mu C$$

3



$$(1) I_{K1} = I_{f1} = 4A$$

$$(2) I_{K2} = I_{f2} = 3A$$

$$(3) (-R_1) I_{K1} + (-R_2) I_{K2} + (R_1 + R_3 + R_2) I_{K3} = E_1 + E_2 + E_3$$

$$-10 \cdot 4 - 10 \cdot 3 + 30 I_{K3} = 80 \Rightarrow I_{K3} = \frac{150}{30} = 5A$$

$$I_1 = I_{K3} = 5A$$

$$P_{E1} = E_1 I_1 = 10 \cdot 5 = 50W$$

4) a) $\bar{U} = \frac{100}{\sqrt{2}} e^{-j\pi/4} = 50\sqrt{2} e^{-j\pi/4} = (50 - j50) V$

$$\bar{Z}_{23} = \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3} = \frac{j20 \cdot 20}{20 + j20} = \frac{j20}{1+j} \frac{1-j}{1-j} = \frac{20 + j20}{2} = 10 + j10 \Omega$$

$$\bar{Z}_{23} = (10 + j10) \Omega = 10\sqrt{2} e^{j\pi/4} \Omega$$

$$\bar{Z}_{123} = \bar{Z}_1 + \bar{Z}_{23} = -j20 + 10 + j10 = (10 - j10) \Omega = 10\sqrt{2} e^{-j\pi/4} \Omega$$

$$\bar{I}_1 = \frac{\bar{U}}{\bar{Z}_{123}} = \frac{50\sqrt{2} e^{-j\pi/4}}{10\sqrt{2} e^{-j\pi/4}} = 5 A \quad \bar{U}_{23} = \bar{Z}_{23} \bar{I}_1 = (50 + j50) V = 50\sqrt{2} e^{j\pi/4} V$$

$$\bar{I}_2 = \frac{\bar{U}_{23}}{\bar{Z}_2} = \frac{50 + j50}{j20} = (2,5 - j2,5) A = 2,5\sqrt{2} e^{-j\pi/4} A$$

$$\bar{I}_3 = \frac{\bar{U}_{23}}{\bar{Z}_3} = \frac{50 + j50}{20} = (2,5 + j2,5) A = 2,5\sqrt{2} e^{j\pi/4} A$$

b) $\bar{U}_1 = \bar{U} - \bar{U}_{23} = \bar{Z}_1 \bar{I}_1 = -j100 V = 100 e^{-j\pi/2} V$

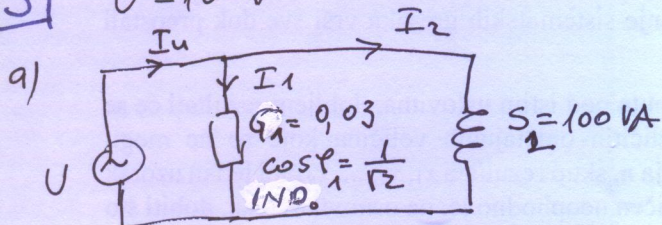
$$u_1(t) = 100\sqrt{2} \cos(\omega t - \pi/2) V$$

c) $\bar{S}_{23} = \bar{Z}_{23} \bar{I}_1^2 = (10 + j10) \cdot 5^2 = (250 + j250) VA = 250\sqrt{2} e^{j\pi/4} VA$

$$P = 250 W, Q = 250 VAR, S = 250\sqrt{2} VA, \cos \varphi = \cos \pi/4 = \frac{1}{\sqrt{2}}$$

5

$$U = 100 V$$



$$Y_1 = \frac{G_1}{\cos \varphi_1} = 0,03\sqrt{2} S$$

$$I_1 = Y_1 U = 3\sqrt{2} A$$

$$I_2 = \frac{S_2}{U} = \frac{100}{100} = 1 A$$

b) $P_U = P_1 + P_2 = U I_1 \cos \varphi_1 + 0 = 3\sqrt{2} \cdot 100 \cdot \frac{1}{\sqrt{2}} = 300 W$

$$Q_2 = \sqrt{S_2^2 - P_2^2} = S_2 = 100 VAR$$

$$Q_1 = U I_1 \sin \varphi_1 = 100 \cdot 3\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 300 VAR \quad \left(\sin \varphi_1 = \sqrt{1 - \cos^2 \varphi_1} = \frac{1}{\sqrt{2}} \right)$$

↑
INDUKTIVAN
 $\varphi_1 > 0, Q_1 > 0$

$$Q_U = Q_1 + Q_2 = 400 VAR$$

$$S_U = \sqrt{P_U^2 + Q_U^2} = \sqrt{300^2 + 400^2} = 100 \sqrt{3^2 + 4^2} = 500 VA$$

c) $I_U = \frac{S_U}{U} = \frac{500}{100} = 5 A$

d) $\bar{Y}_1 = G_1 + jB_1 = (0,03 - j0,03) S$

$$B_1 = -Y_1 \sin \varphi_1 = -0,03 \cdot \sqrt{2} \cdot \frac{1}{\sqrt{2}} = -0,03 S$$

↑
INDUKTIVAN: $B_1 < 0$

KALOM: $\bar{Y}_2 = 0 + jB_2 = jB_2, Y_2 = |B_2| \cdot \frac{I_2}{U} = 0,01 S \Rightarrow \bar{Y}_2 = -j0,01 S$

PARALLELNA VEZA:

$$\bar{Y}_U = \bar{Y}_1 + \bar{Y}_2 = 0,03 - j0,03 - j0,01$$

$$\bar{Y}_U = (0,03 - j0,04) S$$