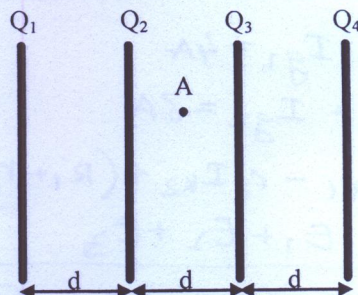


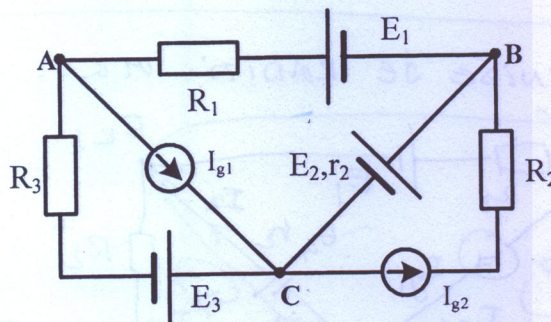
Elektrotehnika

pismeni ispit, 2.10.2025.

1. Četiri veoma velike ravnomerno naelektrisanе metalne ploče, jednakih površina $S = 100 \text{ m}^2$, nalaze se u sredini čija je dielektrična konstanta $\varepsilon = 2 \cdot 10^{-10} \text{ F/m}$, na međusobnom rastojanju od $d = 10 \text{ m}$, kao što je prikazano na Slici 1. Ploče su opterećene količinama naelektrisanja: $Q_1 = -100 \text{ nC}$, $Q_2 = 200 \text{ nC}$, $Q_3 = 300 \text{ nC}$ i $Q_4 = -400 \text{ nC}$. Usvojiti proizvoljan koordinatni sistem, a zatim odrediti i nacrtati vektor električnog polja u tački A. (8 poena)



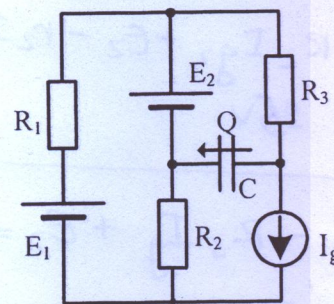
Slika 1



Slika 2

2. U kolu na Slici 2 poznate su vrednosti parametara elemenata: $R_1 = R_2 = R_3 = 5 \Omega$, $E_1 = 5 \text{ V}$, $E_2 = 20 \text{ V}$, $E_3 = 15 \text{ V}$, $r_2 = 5 \Omega$, $I_{g1} = 4 \text{ A}$ i $I_{g2} = 3 \text{ A}$. Rešiti kolo primenom metode potencijala čvorova ili metode konturnih struja, a nakon toga odrediti snagu generatora I_{g2} . (12 poena)

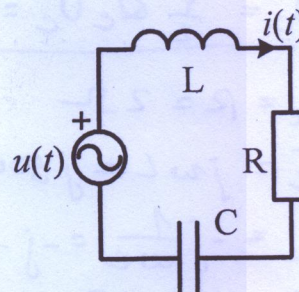
3. Na Slici 3 poznato je $R_1 = 10 \Omega$, $R_2 = R_3 = 5 \Omega$, $E_1 = E_2 = 30 \text{ V}$, $I_g = 3 \text{ A}$ i $C = 5 \mu\text{F}$. Odrediti količinu naelektrisanja i elektrostatičku energiju kondenzatora. (6 poena)



Slika 3

4. Na Slici 4 je prikazano kolo naizmenične struje poznatih parametara elemenata: $R = 2 \Omega$, $L = 8 \text{ mH}$ i $C = 1 \text{ mF}$. Kroz kolo protiče struja čiji je vremeski oblik: $i(t) = \sqrt{2} \cos(500t - \pi/2) \text{ A}$.

- Odrediti kompleksne izraze struje u kolu i napona na otporniku, kalemu i kondenzatoru (sa usaglašenim referentnim smerovima); (4 poena)
- Skicirati fazorski dijagram struje i napona na svim elementima; (4 poena)
- Odrediti vremeski oblik napona generatora $u(t)$. (4 poena)

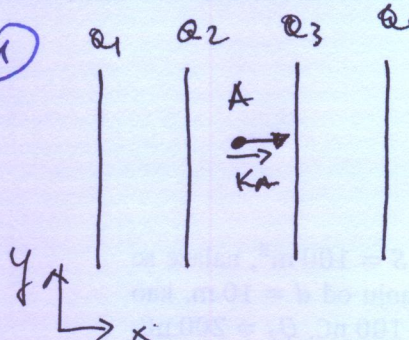


Slika 4

5. Na naponski generator efektivne vrednosti $U = 100 \text{ V}$, priključena je paralelna veza pretežno induktivnog potrošača, čija je admitansa $0.4 \cdot \sqrt{2} \text{ S}$ i faktor snage $1/\sqrt{2}$, i kondenzatora, čija je prividna snaga 1000 VA . Odrediti:

- efektivne vrednosti struja pojedinačnih potrošača; (3 poena)
- ukupnu aktivnu, reaktivnu i prividnu snagu celokupnog potrošača (paralelne veze); (5 poena)
- efektivnu vrednost struje koju celokupni potrošač uzima iz generatora; (2 poena)
- kompleksnu admitansu celokupnog potrošača. (2 poena)

①



$$\vec{K}_A = \vec{K}_{A1} + \vec{K}_{A2} + \vec{K}_{A3} + \vec{K}_{A4}$$

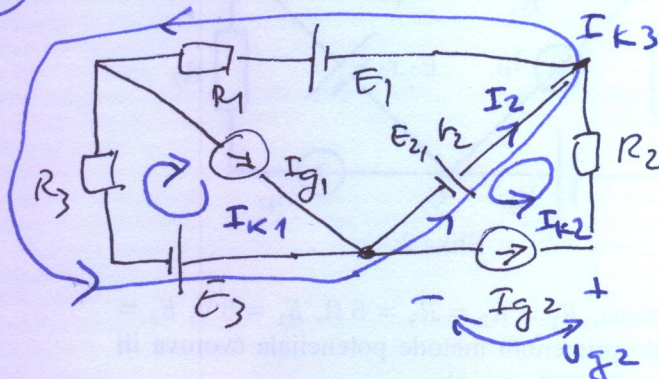
$$\vec{K}_A = \frac{\sigma_1}{2\epsilon} \vec{e} + \frac{\sigma_2}{2\epsilon} \vec{e} + \frac{\sigma_3}{2\epsilon} (-\vec{e}) + \frac{\sigma_4}{2\epsilon} (-\vec{e})$$

$$\vec{K}_A = \left(\frac{Q_1/S}{2\epsilon} + \frac{Q_2/S}{2\epsilon} - \frac{Q_3/S}{2\epsilon} - \frac{Q_4/S}{2\epsilon} \right) \vec{e}$$

$$\vec{K}_A = \frac{Q_1 + Q_2 - Q_3 - Q_4}{2\epsilon S} \vec{e} = \frac{200 \cdot 10^{-9}}{2 \cdot 2 \cdot 10^{-10} \cdot 100} \vec{e}$$

$$\vec{K}_A = 5 \vec{e} \frac{V}{m}$$

② Povoljnije je raditi mks.



$$P_{I_{g2}} = U_{g2} \cdot I_{g2} = 25 \cdot 3 = 75 \text{ W}$$

$$U_{g2} = R_2 I_{g2} + E_2 - r_2 I_2 = 5 \cdot 3 + 20 - 5 \cdot 2$$

$$U_{g2} = 25 \text{ V}$$

(1) $I_{K1} = I_{g1} = 4 \text{ A}$

(2) $I_{K2} = I_{g2} = 3 \text{ A}$

(3) $-R_3 I_{K1} - r_2 I_{K2} + (R_1 + r_2 + R_3) I_{K3} = E_1 + E_2 + E_3$

$$\Rightarrow -5 \cdot 4 - 5 \cdot 3 + (5 + 5 + 5) I_{K3} = 5 + 20 + 15$$

$$15 I_{K3} = 40 + 15 = 55$$

$$I_{K3} = \frac{55}{15} = 3.67 \text{ A}$$

$$I_2 = I_{K3} - I_{K2} = 3.67 - 3 = 0.67 \text{ A}$$

③ $U_c = -R_3 I_g + E_2 = -5 \cdot 3 + 30 = 15 \text{ V}$

$$Q_c = C U_c = 5 \mu\text{F} \cdot 15 = 75 \mu\text{C}$$

$$W_c = \frac{1}{2} Q_c U_c = \frac{1}{2} 75 \cdot 15 = 562.5 \mu\text{J}$$

④ a) $\bar{Z}_R = R = 2 \Omega$

$$\bar{Z}_L = j\omega L = j500 \cdot 8 \cdot 10^{-3} = j4 \Omega$$

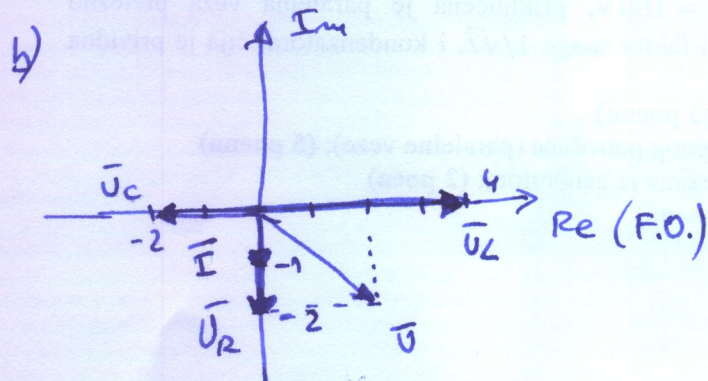
$$\bar{Z}_C = -j \frac{1}{\omega C} = -j \frac{1}{500 \cdot 10^{-3}} = -j2 \Omega$$

$$\bar{I}_a = 1 \cdot e^{-j\pi/4} = -j \text{ A}$$

$$\bar{U}_R = \bar{Z}_R \bar{I} = -j2 \text{ V} = 2 e^{-j\pi/2} \text{ V}$$

$$\bar{U}_L = \bar{Z}_L \bar{I} = 4 \text{ V} = 4 e^{j0} \text{ V}$$

$$\bar{U}_C = \bar{Z}_C \bar{I} = -2 \text{ V} = 2 e^{j\pi} \text{ V}$$



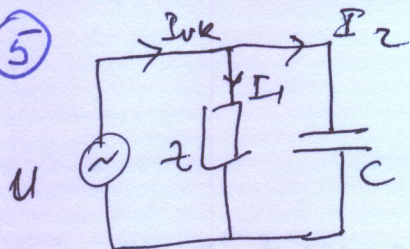
c) $\bar{U} = \bar{U}_R + \bar{U}_L + \bar{U}_C$

$$\bar{U} = -j2 + 4 - 2 = 2 - j2 \text{ V}$$

$$\bar{U} = 2\sqrt{2} e^{-j\pi/4} \text{ V}$$

$$u(t) = 4 \cos(500t - \pi/4) \text{ V}$$

5



a)

$$Y = 0,4\sqrt{2} \text{ S} \Rightarrow Z = \frac{1}{Y} = \frac{1}{0,4\sqrt{2}} = \frac{10\sqrt{2}}{4} = \frac{5}{2}\sqrt{2} \Omega$$

$$\cos \varphi = \frac{1}{\sqrt{2}}$$

$$I_1 = \frac{U}{Z} = \frac{100 \cdot 4}{5\sqrt{2}} = 40\sqrt{2} \text{ A}$$

$$(\approx 40 \text{ V})$$

$$I_2 = \frac{S_2}{U} = \frac{1000}{100} = 10 \text{ A}$$

b) $P_1 = R I_1^2 = Z I^2 \cos \varphi = \frac{5}{2}\sqrt{2} \cdot (40)^2 \cdot 2 \cdot \frac{1}{\sqrt{2}} = \frac{10}{4} \cdot 10^4 \cdot 40 = 4000 \text{ W}$

$$Q_1 = X I_1^2 = Z I^2 \sin \varphi = 4000 \text{ VAR}$$

$$\sin \varphi = +\sqrt{1 - \cos^2 \varphi} = \frac{1}{\sqrt{2}}$$

$$P_2 = 0 \text{ W} \text{ (kondenzator nema aktivnu snagu)}$$

$$Q_2 = -\sqrt{S_2^2 - P_2^2} = -S_2 = -1000 \text{ VA}$$

$$P_{uk} = P_1 + P_2 = 4000 \text{ W}$$

$$Q_{uk} = Q_1 + Q_2 = 4000 - 1000 = 3000 \text{ VAR}$$

$$S_{uk} = \sqrt{P_{uk}^2 + Q_{uk}^2} = 5000 \text{ VA}$$

c) $I_{uk} = \frac{S_{uk}}{U} = \frac{5000}{100} = 50 \text{ A}$

d) $\vec{S}_{uk} = P_{uk} + j Q_{uk} =$

$$\vec{S}_{uk} = \frac{U^2}{Z^*} = \vec{Y}^* U^2$$

$$\vec{Y} = \vec{Y}^{**} = \frac{\vec{S}_{uk}^*}{U^2} = \frac{4000 - j3000}{(100)^2} = (0,4 - j0,3) \text{ S}$$