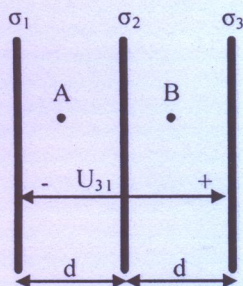


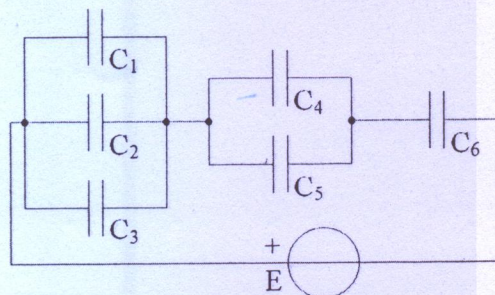
Elektrotehnika

pismeni ispit, 20.10.2025.

1. Tri veoma velike ravnomerno naelektrisanе metalne ploče nalaze se u sredini čija je dielektrična konstanta $\epsilon = 10^{-8} \text{ F/m}$, na međusobnom rastojanju od $d = 5 \text{ m}$ kao što je prikazano na Slici 1. Poznate su površinske gustine naelektrisanja ploče $\sigma_1 = 40 \mu\text{C/m}^2$, $\sigma_2 = 20 \mu\text{C/m}^2$ i $\sigma_3 = 60 \mu\text{C/m}^2$. Odrediti i skicirati vektor električnog polja u tačkama A i B, a zatim odrediti napon U_{31} . (8 poena)



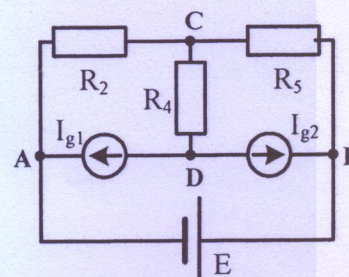
Slika 1



Slika 2

2. U kolu na Slici 2 poznate su kapacitivnosti $C_1 = C_4 = C_5 = C = 6 \mu\text{F}$, $C_2 = 2C$, $C_3 = C_6 = 3C$, i elektromotorna sila idealnog naponskog generatora $E = 12 \text{ V}$. Pre povezivanja, svi kondenzatori su bili neopterećeni. Odrediti naelektrisanje na C_2 , napon na C_6 i ukupnu energiju paralelne veze C_4 i C_5 . (8 poena)

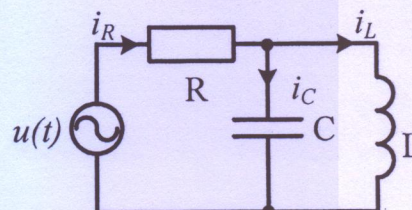
3. U kolu na Slici 3 poznate su vrednosti parametara elemenata: $R_2 = R_4 = R_5 = 10 \Omega$, $E = 50 \text{ V}$, $I_{g1} = 4 \text{ A}$ i $I_{g2} = 3 \text{ A}$. Rešiti kolo primenom metode potencijala čvorova ili metode konturnih struja, a nakon toga odrediti snagu generatora I_{g1} . (10 poena)



Slika 3

4. Na Slici 4 je prikazano kolo naizmenične struje koje čine naponski generator $u(t) = 200 \cos(200t - \pi/4) \text{ V}$ i tri elementa poznatih parametara: $R = 20 \Omega$, $L = 50 \text{ mH}$ i $C = 0.25 \text{ mF}$.

- Odrediti kompleksne izraze za struje u svim granama kola i skicirati njihov fazorski dijagram. (6 poena)
- Odrediti trenutnu vrednost napona na kondenzatoru. (2 poena)
- Odrediti aktivnu, reaktivnu i prividnu snagu i faktor snage celog potrošača. (4 poena)

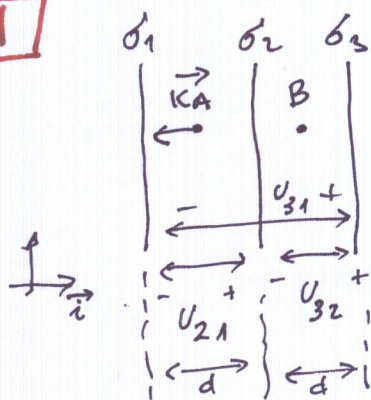


Slika 4

5. Na naponski generator efektivne vrednosti $U = 100 \text{ V}$, priključena je paralelna veza pretežno induktivnog potrošača, konduktanse 0.01 S i faktora snage $1/\sqrt{2}$, i jednog idealnog kondenzatora prividne snage 100 VA . Odrediti:

- efektivne vrednosti struja pojedinačnih potrošača; (3 poena)
- ukupnu aktivnu, reaktivnu i prividnu snagu celokupnog potrošača (paralelne veze); (5 poena)
- efektivnu vrednost struje koju celokupni potrošač uzima iz generatora; (2 poena)
- kompleksnu admitansu celokupnog potrošača. (2 poena)

1



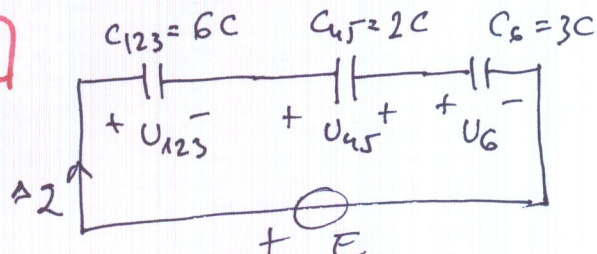
$$\vec{K}_A = \frac{d_1 - d_2 - d_3}{2\epsilon} \vec{i} = -2 \vec{i} \frac{\text{kV}}{\text{m}}$$

$$\vec{K}_B = \frac{d_1 + d_2 - d_3}{2\epsilon} \vec{i} = \vec{0} \frac{\text{V}}{\text{m}}$$

$$U_{31} = U_{32} + U_{21} = \vec{K}_A \cdot (-d \vec{i}) = -2 \vec{i} \cdot 10^3 \cdot 5 \cdot (-\vec{i})$$

$$U_{31} = 10 \text{ kV}$$

2



$$U_{123} + U_{45} + U_6 = E$$

$$\Delta Q \left(\frac{1}{6C} + \frac{1}{2C} + \frac{1}{3C} \right) = E$$

$$\Delta Q = CE = 72 \mu\text{C}$$

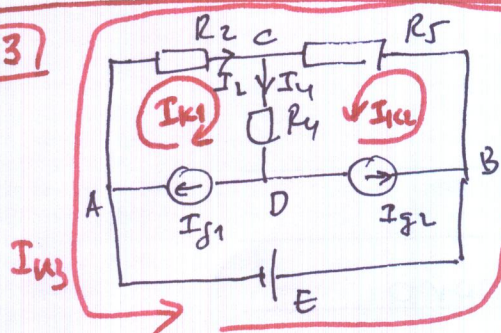
$$U_{123} = \frac{\Delta Q}{6C} = \frac{E}{6} = 2 \text{ V}$$

$$U_6 = \frac{\Delta Q}{3C} = \frac{E}{3} = 4 \text{ V} \quad W_{45} = \frac{\Delta Q^2}{2C_{45}} = \frac{CE^2}{4}$$

$$Q_2 = C_2 U_{123} = \frac{CE}{3} = 24 \mu\text{C}$$

$$W_{45} = 216 \mu\text{J}$$

3



$$\text{MKS (1) } I_{g1} = I_{K1} = 4 \text{ A}$$

$$(2) I_{K2} = I_{g2} = 3 \text{ A}$$

$$(3) -R_2 I_{K1} + R_5 I_{K2} + (R_2 + R_5) I_{K3} = E$$

$$I_{K3} = \frac{E + R_2 I_{g1} - R_5 I_{g2}}{R_2 + R_5} = 3 \text{ A}$$

$$I_2 = I_{K1} - I_{K3} = 1 \text{ A} \quad U_{AD} = R_2 I_2 + R_4 I_4 = 80 \text{ V}$$

$$I_4 = I_{K1} + I_{K2} = 7 \text{ A} \quad P_{I_{g1}} = I_{g1} \cdot U_{AD} = 320 \text{ W}$$

5

$$\bar{U} = 100\sqrt{2} e^{j\varphi_4} \text{ V}$$

$$\bar{Z}_R = R = 20 \Omega$$

$$\bar{Z}_L = j\omega L = j10 \Omega$$

$$\bar{Z}_C = -j\frac{1}{\omega C} = -j20 \Omega$$

$$\bar{Z}_{LC} = \frac{\bar{Z}_L \bar{Z}_C}{\bar{Z}_L + \bar{Z}_C} = j20 \Omega$$

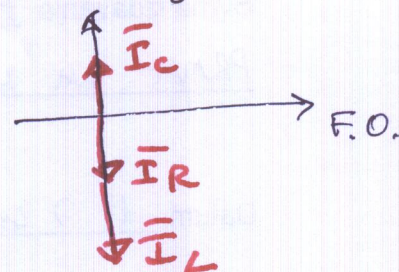
$$\bar{Z}_{RLC} = \bar{Z}_R + \bar{Z}_{LC} = 20 + j20 = 20\sqrt{2} e^{j\varphi_4} \Omega$$

$$\bar{I}_R = \frac{\bar{U}}{\bar{Z}_{RLC}} = \frac{100\sqrt{2} e^{j\varphi_4}}{20\sqrt{2} e^{j\varphi_4}} = 5 e^{-j\varphi_4} = -j5 \text{ A}$$

$$\bar{U}_{LC} = \bar{Z}_{LC} \cdot \bar{I}_R = 100 \text{ V}$$

$$\bar{I}_L = \frac{\bar{U}_{LC}}{\bar{Z}_L} = -j10 \text{ A}$$

$$\bar{I}_C = \frac{\bar{U}_{LC}}{\bar{Z}_C} = j5 \text{ A}$$



$$b) \bar{U}_C = \bar{U}_{LC} = 100 = 100 \cdot e^{j0} \text{ V} \Rightarrow u_C(t) = 100\sqrt{2} \cos(200t) \text{ V}$$

$$c) \bar{S} = \bar{Z}_{RLC} I_R^2 = (20 + j20) \cdot 25 = (500 + j500) \text{ VA} \Rightarrow P = 500 \text{ W}, Q = 500 \text{ VAR}$$

$$S = 500\sqrt{2} \text{ VA}, \cos\varphi = \frac{P}{S} = \frac{1}{\sqrt{2}}$$

6

$$a) \text{ PRVI POTROŠAČ: } G_1 = 0,01 \text{ S} \Rightarrow Y_1 = \frac{G_1}{\cos\varphi_1} = 0,01\sqrt{2} \text{ S} \Rightarrow I_1 = Y_1 U = \sqrt{2} \text{ A}$$

$$\text{DRUGI POTROŠAČ (C): } I_2 = \frac{S_2}{U} = 1 \text{ A}$$

$$b) P_1 = U I_1 \cos\varphi_1 = 100 \text{ W}$$

$$P_2 = 0$$

$$Q_1 = U I_1 \sin\varphi_1 = 100 \text{ VAR}$$

$$Q_2 = -S_2 = -100 \text{ VAR}$$

$$(\sin\varphi_1 = +\sqrt{1 - \cos^2\varphi_1} = \frac{1}{\sqrt{2}}) \text{ INDUKTIVAN}$$

$$\text{KAPACITIVNOST}$$

$$P_U = P_1 + P_2 = 100 \text{ W}$$

$$Q_U = Q_1 + Q_2 = 0 \text{ VAR}$$

$$S_U = \sqrt{P_U^2 + Q_U^2} = 100 \text{ VA}$$

$$c) I_U = \frac{S_U}{U} = \frac{100 \text{ VA}}{100 \text{ V}} = 1 \text{ A}$$

$$d) G_1 = 0,01 \text{ S}$$

$$B_1 = -Y_1 \sin \varphi_1 = 0,01 \text{ S}$$

↑
INDUKTIVAN

$$\bar{Y}_1 = G_1 + jB_1 = (0,01 - j0,01) \text{ S}$$

$$G_2 = 0 \text{ S}$$

$$B_2 = -\frac{Q_2}{U^2} = \frac{S_2}{U^2} = 0,01 \text{ S} (> 0 \text{ KAP.})$$

$$\bar{S}_2 = \bar{Y}_2^* U^2 = (G_2 - jB_2) U^2 = \underbrace{G_2 U^2}_0 - \underbrace{jB_2 U^2}_{jQ_2}$$

$$\bar{Y}_U = \bar{Y}_1 + \bar{Y}_2 = (0,01 + j0) \text{ S}$$

$$\text{ili } \bar{S}_U = \bar{Y}^* U^2 \Rightarrow \bar{Y}^* = \frac{\bar{S}_U}{U^2} \Rightarrow \bar{Y} = \frac{\bar{S}_U^*}{U^2} = \frac{100 + j0}{100^2}$$

$$\bar{Y} = 0,01 + j0 \text{ S}$$