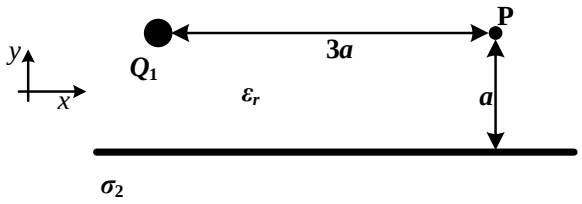
Elektrotehnika - pismeni ispit

27. februar 2026.

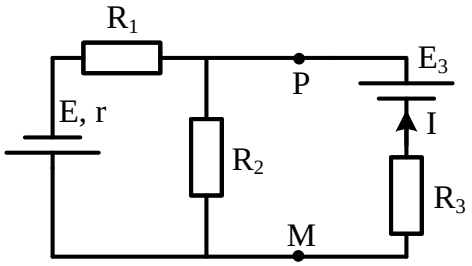
1. Tačkasto naelektrisanje $Q_1 = -Q < 0$ i veoma velika, ravnomerno naelektrisana površ, površinske gustine naleketrisanja $\sigma_2 = \sigma > 0$ nalaze se u dielektriku relativne dielektrične permitivnosti ε_r kao na Slici 1.

a) Odrediti i skicirati **vektor** elektičnog polja u tački P. **(4 poena)**

b) Odrediti i skicirati **vektor** sile kojom površ deluje na tačkasto naelektisanje. **(3 poena)**



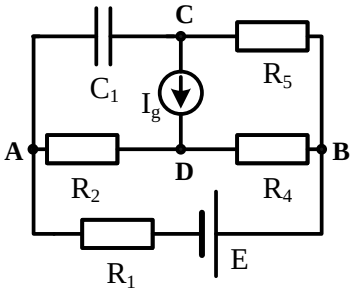
Slika 1



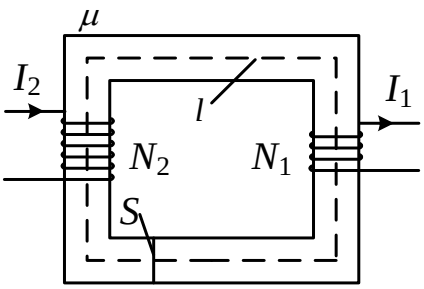
Slika 2

2. U kolu prikazanom na Slici 2, primenom Tevenenove teoreme odrediti intenzitet struje I i napon U_{MP}. Poznato je: $E = 20\,\mathrm{V}$, $r = 5\,\Omega$, $E_3 = 20\,\mathrm{V}$, $R_1 = 5\,\Omega$, $R_2 = 10\,\Omega$, $R_3 = 10\,\Omega$. **(10 poena)**

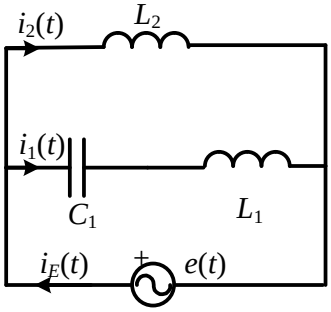
3. U kolu sa Slike 3 potrebno je odrediti energiju kondenzatora. Poznato je: $E = 12\,\mathrm{V}$, $I_g = 6\,\mathrm{A}$, $R_1 = R_2 = R_4 = R_5 = 4\,\Omega$, $C_1 = 20\,\mu\mathrm{F}$. **(8 poena)**



Slika 3



Slika 4



Slika 5

4. U kolu na Slici 4 prikazano je magnetno kolo sa dva namotaja. Kroz namotaj sa $N_1 = 25$ navojaka protiče struja $I_1 = 12\,\mathrm{A}$, a kroz namotaj sa $N_2 = 50$ navojaka struja $I_2 = 10\,\mathrm{A}$. Jezgro je površine poprečnog preseka $S = 5\,\mathrm{cm}^2$, dužine srednje linije $l = 20\,\mathrm{cm}$ i magnetne permeabilnosti $\mu = 2 \cdot 10^{-4}\,\mathrm{H/m}$.

a) Odrediti fluks vektora magnetne indukcije u jezgru. **(6 poena)**

b) Odrediti energiju magnetnog polja u jezgru. **(4 poena)**

c) Odrediti koju vrednost bi trebalo da ima struja u drugom namotaju, $I_2' = ?$, tako da energija magnetnog polja bude jednaka nuli. **(2 poena)**

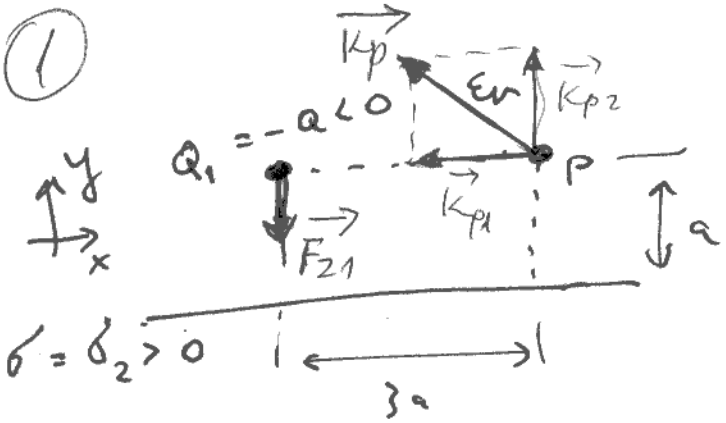
5. U kolu na Slici 5 poznato je: $e(t) = 200\sqrt{2}\cos(500t - \pi)\,\mathrm{V}$, $C_1 = 80\,\mu\mathrm{F}$, $L_1 = 40\,\mathrm{mH}$, $L_2 = 20\,\mathrm{mH}$. Odrediti:

a) kompleksne struje u granama i nacrtati fazorski dijagram, **(5 poena)**

b) vremenski oblik napona na kondenzatoru, **(2 poena)**

c) aktivnu, reaktivnu i prividnu snagu i faktor snage celokupnog potrošača, **(4 poena)**

d) kompleksnu admitansu potrošača. **(2 poena)**

① 

a) $\vec{K_p} = \vec{K_{p1}} + \vec{K_{p2}}$

$\vec{K_{p1}} = \frac{Q_1}{4\pi\varepsilon_0\varepsilon_r (3a)^2}\vec{i}$

$\vec{K_{p1}} = \frac{-Q}{36\pi\varepsilon_0\varepsilon_r a^2}\vec{i}$

$\vec{K_{p2}} = \frac{\sigma_2}{2\varepsilon_0\varepsilon_r}\vec{j} = \frac{\sigma}{2\varepsilon_0\varepsilon_r}\vec{j}$

$\vec{K_p} = \frac{-Q}{36\pi\varepsilon_0\varepsilon_r a^2}\vec{i} + \frac{\sigma}{2\varepsilon_0\varepsilon_r}\vec{j}$

b) $\vec{F_{21}} = \vec{K_2}\cdot Q_1 = \frac{\sigma_2 Q_1}{2\varepsilon_0\varepsilon_r}\vec{j}$

$\vec{F_{21}} = -\frac{\sigma Q}{2\varepsilon_0\varepsilon_r}\vec{j}$

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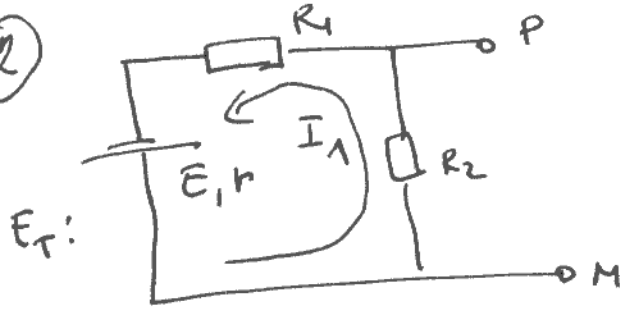
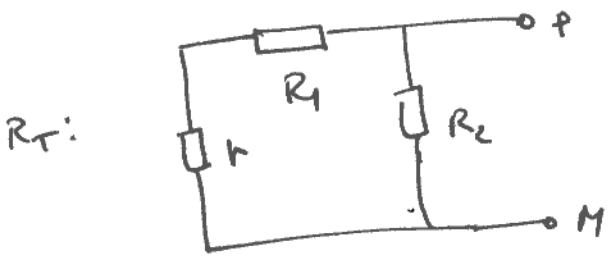
② E_T: 

$E_T = U_{MP}^{0V}$

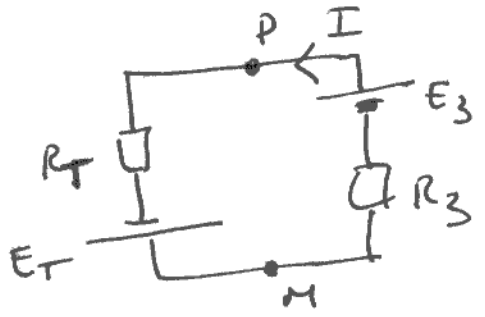
$I_1 = \frac{E}{r+R_1+R_2} = \frac{20}{5+5+10} = 1A$

$E_T = R_2 I_1 = 10V$

R_T: 

$R_T = R_{MP} = R_2 \,||\, (R_1+r) = \frac{R_2\cdot(R_1+r)}{r+R_1+R_2}$

$R_T = 5\Omega$



$I = \frac{E_T+E_3}{R_T+R_3} = \frac{10+20}{5+15} = 2A$

$U_{MP} = -E_3 + R_3 I = -20+10\cdot 2 = 0V$

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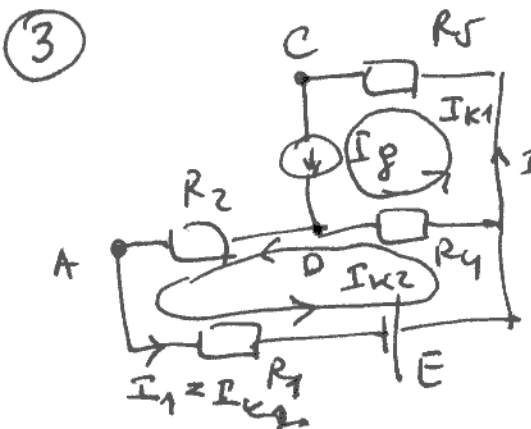
③ 

(1) $I_{K1} = I_g = 6A$

(2) $R_{21} I_{K1} + R_{22} I_{K2} = \sum_{K2} E_i$

$-R_4 I_g + (R_1+R_2+R_4) I_{K1} = E$

$I_{K1} = \frac{E+R_4 I_g}{R_1+R_2+R_4} = \frac{12+4\cdot 6}{4+4+4} = 3A$

$U_{CA} = U_{AC} = R_1 I_1 - E + R_5 I_5 = 24V$

$$W_{C1} = \frac{1}{2} C_1 \cdot U_{C1}^2 = \frac{1}{2} \cdot 20 \cdot 10^{-6} \cdot \underbrace{(24)^2}_{576} = 5760\,\mu J = 5{,}76\,mJ$$

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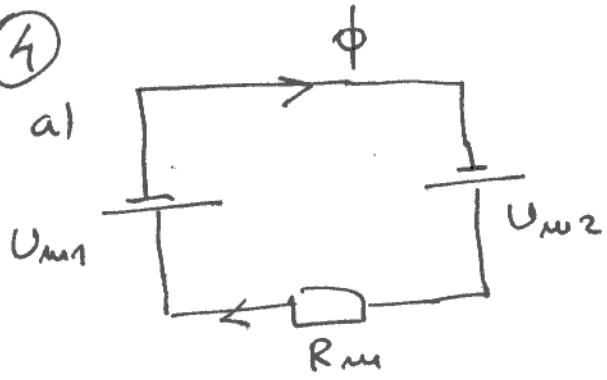
④

a)



$U_{m1} = N_1 I_1$

$U_{m2} = N_2 I_2$

$R_m = \frac{l}{\mu S}$

$S = 5\,cm^2 = 5 \cdot (10^{-2})^2\,m^2$

$S = 5 \cdot 10^{-4}\,m^2$

$l = 20\,cm = 0{,}2\,m$

$$\phi = \frac{U_{m2} - U_{m1}}{R_m} = \frac{N_2 I_2 - N_1 I_1}{\frac{l}{\mu S}}$$

b) $B = \frac{\phi}{S} = \frac{100\mu}{5 \cdot 10^{-4}}$

$$\phi = \frac{50 \cdot 10 - 25 \cdot 12}{\frac{0{,}2}{2 \cdot 10^{-4} \cdot 5 \cdot 10^{-4}}} = \frac{200}{2 \cdot 10^6} = 100\,\mu Wb$$

$B = 0{,}2\,T$

$H = \frac{B}{\mu} = \frac{0{,}2}{2 \cdot 10^{-4}} = 1000\,\frac{A}{m}$

$w_m = \frac{1}{2} B H \Rightarrow W_m = Sl \cdot w_m = \frac{BH \cdot Sl}{2} = \frac{0{,}2 \cdot 10^3 \cdot 5 \cdot 10^{-4} \cdot 0{,}2}{2}$

$W_m = 10\,mJ$

c) $W_m' = 0 \Rightarrow B' = 0,\ H' = 0 \Rightarrow \phi' = 0 \Rightarrow N_1 I_1 = N_2 I_2'$

$I_2' = \frac{N_1}{N_2} I_1 = 6A$

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⑤ a) $\bar{E} = 200 e^{j\frac{\pi}{2}} V = -200\,V$

$\bar{Z}_{L1} = j\omega L_1 = j\,500 \cdot 40m = j20\,\Omega$

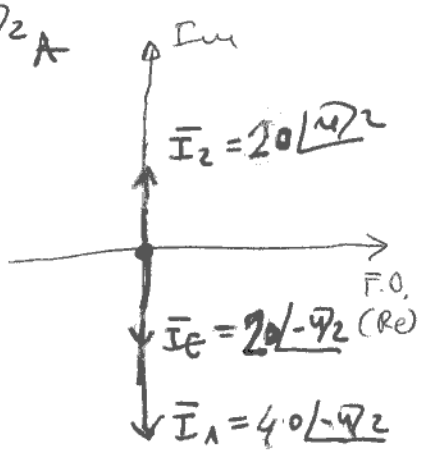
$\bar{Z}_{L2} = j\omega L_2 = j\,500 \cdot 20m = j10\,\Omega$

$\bar{Z}_{C1} = -j\frac{1}{\omega C_1} = -j\frac{1}{500 \cdot 80\mu} = -j\frac{10^6}{40 \cdot 10^3} = -j25\,\Omega$

$\bar{I}_1 = \frac{\bar{E}}{\bar{Z}_{L1} + \bar{Z}_{C1}} = \frac{-200}{j20 - j25} = \frac{-200}{-j5} = -j40\,A = 40 e^{-j\frac{\pi}{2}} A$

$\bar{I}_2 = \frac{\bar{E}}{\bar{Z}_{L2}} = \frac{-200}{j10} = j20\,A = 20 e^{j\frac{\pi}{2}} A$

$\bar{I}_E = \bar{I}_1 + \bar{I}_2 = -j40 + j20 = -j20\,A$



b) $\bar{U}_{c1} = \bar{Z}_{c1} \cdot \bar{I}_1 = (-j25) \cdot (-j40) = -1000\,V = 1000 e^{j\pi}\,V$

$u_{c1}(t) = 1000\sqrt{2}\cos(500t - \pi)\,V$

c) $\bar{S}_p = \bar{S}_g = \bar{E}\bar{I}_E^* = -200 \cdot (-j20)^* = -200 \cdot (j20)$

$\bar{S}_p = (-j4000)\,VA$

$P = 0\,W$, $Q = -4\,kVAR$, $S = 4\,kVA$

$\cos\varphi = \frac{P}{S} = 0$

d) $\bar{Y}_e = \bar{Y}_1 + \bar{Y}_2$

$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{\bar{Z}_{L1} + \bar{Z}_{C1}} = \frac{1}{-j5} = j0,2\,S$

$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{\bar{Z}_{L2}} = \frac{1}{j10} = -j0,1\,S$

$\bar{Y}_e = j0,2 - j0,1 = j0,1\,S$